Ricardian Models & Key Facts

Ambrose W

1 Chapter 2: World Trade – An Overview

1.1 The Gravity Model

The gravity model postulates an empirical relationship between the size of a country's economy and the volume of its imports and exports.

1.1.1 The Equation

Derived by analogy from Newton's Law of Universal Gravitation, the value of trade between two countries is proportional to the product of their GDPs and inversely proportional to the distance between them.

$$T_{ij} = A \cdot Y_i^a \cdot \frac{Y_j^b}{D_{ij}^c}$$

Where:

- T_{ij}: Value of trade between country i and country j
- A: Constant
- Y_i: GDP of country i
- Y_j: GDP of country j
- D_{ij}: Distance between country i and country j
- a, b, c: Constants to be estimated

Economic Intuition:

1. **Size Effect (Y):** Larger economies have higher income (spend more on imports) and produce a wider variety of goods (attract more foreign spending).
2. **Distance Effect (D):** Distance serves as a proxy for transport costs and other barriers. Typically, a 1% increase in distance results in a 0.7%–1.0% decrease in trade.

1.1.2 Anomalies and Impediments

While the model fits data well generally, anomalies exist due to factors beyond GDP and physical distance:

- **Cultural Affinity:** Ireland trades more with the U.S. than predicted due to cultural ties and Multinational Corporation (MNC) registrations.
- **Geography/Transport Hubs:** Belgium and the Netherlands trade more due to location (mouth of the Rhine) and port infrastructure (Rotterdam, Antwerp).
- **Borders:** Political borders significantly reduce trade. Even with free trade agreements (like USMCA/NAFTA), trade between Canadian provinces is significantly larger than trade between Canadian provinces and U.S. states of equal distance.

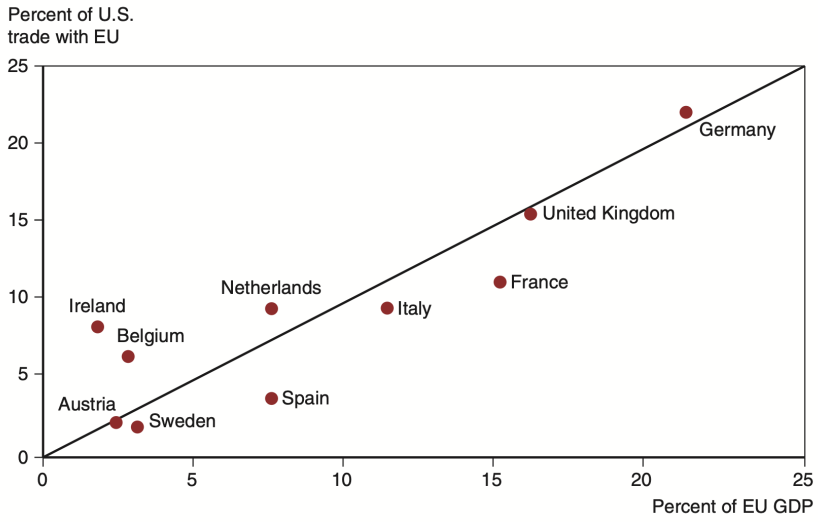


Figure 1: Relationship between Economic Size and Trade Volume (US-EU)

1.2 The Changing Pattern of World Trade

1.2.1 Evolution of Trade Patterns

- **Victorian Globalization (1840–1914):** Driven by steamships and telegraphs.
- **Interwar Period:** Trade collapsed due to wars and protectionism.
- **Post-1970 (Hyper-Globalization):** Soaring trade driven by "Vertical Disintegration" (fragmentation of production stages across countries).

1.2.2 Composition of Trade

Historically (e.g., 1910), developed countries (UK) exported manufactures and imported primary commodities. Today, manufactured goods dominate world trade (approx. 70%).

- **Developing Economies:** Have shifted significantly from exporting agricultural/mining products (1960s) to manufactured goods (present day).

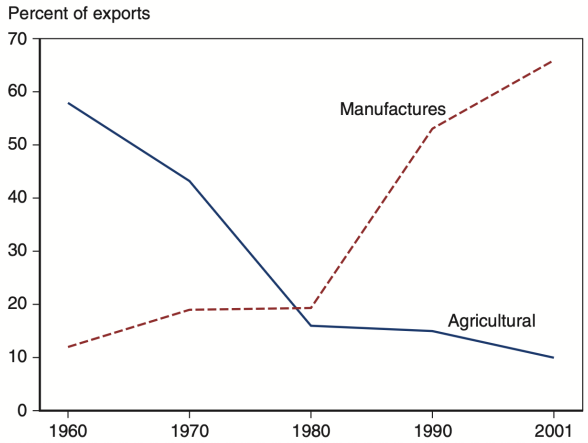


Figure 2: Shift in Developing Economy Exports (Primary to Manufacturing)

1.2.3 Service Offshoring

Trade in services is becoming increasingly important. The key distinction is no longer just "manufacturing vs. agriculture" but **tradable vs. non-tradable** services.

- **Tradable:** Services that can be performed remotely (e.g., professional services).
- **Non-tradable:** Services requiring physical presence (e.g., retail, personal services).

Currently, roughly 60% of U.S. employment is in non-tradable sectors.

2 Chapter 3: The Ricardian Model

2.1 Fundamental Concepts

- **Why Trade?**
  1. **Intrinsic Differences:** Productivity differences (Ricardian) or factor endowment differences (Heckscher-Ohlin).
  2. **Economies of Scale:** Producing a limited range of goods more efficiently.
- **Opportunity Cost:** The cost of the best alternative forgone. Trade allows countries to rearrange production based on comparative rather than absolute costs.
- **Comparative Advantage:** A country has a comparative advantage in a good if the opportunity cost of producing that good (in terms of other goods) is lower than in other countries.

2.2 The One-Factor Ricardian Model

2.2.1 Assumptions

- Two countries: Home and Foreign (*).
- Two goods: Wine (W) and Cheese (C).
- One factor of production: Labor (L).
- Constant Unit Labor Requirements: a_{LC} and a_{LW} (hours to produce one unit).
- Perfect competition and labor mobility within countries.

2.2.2 Production Possibilities Frontier (PPF)

The PPF is determined by the limits of the labor supply:

$$a_{LC}Q_C + a_{LW}Q_W \leq L$$

- **Slope of PPF:** $-\frac{a_{LC}}{a_{LW}}$. The absolute value represents the opportunity cost of Cheese in terms of Wine.

2.2.3 Prices and Wages in Autarky

In a closed economy (Autarky), prices reflect opportunity costs. Assuming perfect labor mobility, labor moves to the sector paying the higher wage. For both goods to be produced:

$$w_C = w_W \Rightarrow \frac{P_C}{a_{LC}} = \frac{P_W}{a_{LW}}$$

Thus, the relative price in autarky is:

$$\frac{P_C}{P_W} = \frac{a_{LC}}{a_{LW}}$$

2.2.4 Trade and Specialization

Assume Home has a comparative advantage in Cheese:

$$\frac{P_C}{P_W} = \frac{a_{LC}}{a_{LW}} < \frac{a^*_{LC}}{a^*_{LW}} = \frac{P^\star_C}{P^\star_W}$$

This implies Home has a lower opportunity cost of producing Cheese. Note that under autarky: relative price of cheese to wine reflects the opportunity cost of cheese in terms of wine in each country

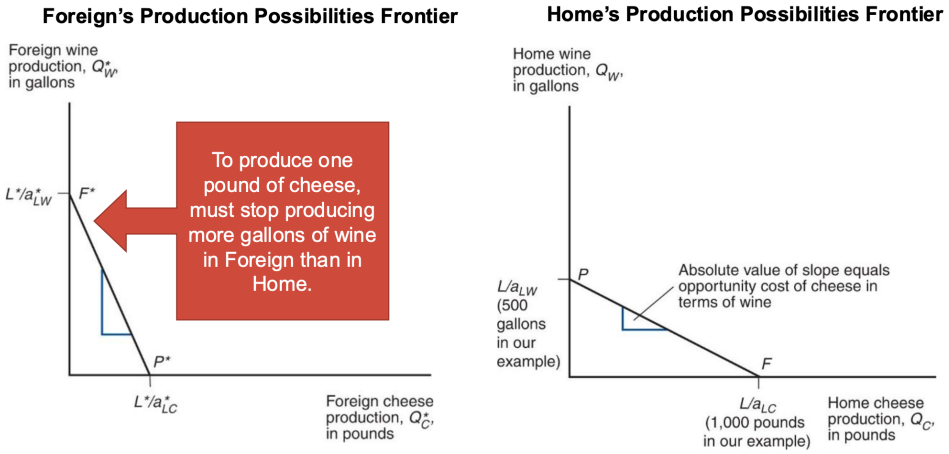


Figure 3: Production Possibilities Frontiers: Home and Foreign

Determining the Equilibrium Relative Price: We utilize a General Equilibrium Analysis using Relative Supply (RS) and Relative Demand (RD) of cheese.

$$RS = \frac{Q_C + Q^*_C}{Q_W + Q^*_W}$$

The RS curve is a "step function" with five segments based on the world relative price P_C/P_W:

1. $\frac{P_C}{P_W} < \frac{a_{LC}}{a_{LW}}$: Home and Foreign specialize in Wine ($RS = 0$).
2. $\frac{P_C}{P_W} = \frac{a_{LC}}{a_{LW}}$: Home is indifferent; Foreign specializes in Wine (Flat segment).
3. $\frac{a_{LC}}{a_{LW}} < \frac{P_C}{P_W} < \frac{a^*_{LC}}{a^*_{LW}}$: **Complete Specialization**. Home produces Cheese; Foreign produces Wine.
$$RS = \frac{L/a_{LC}}{L^*/a^*_{LW}}$$
4. $\frac{P_C}{P_W} = \frac{a^*_{LC}}{a^*_{LW}}$: Home specializes in Cheese; Foreign is indifferent (Flat segment).
5. $\frac{P_C}{P_W} > \frac{a^*_{LC}}{a^*_{LW}}$: Home and Foreign specialize in Cheese ($RS \to \infty$).

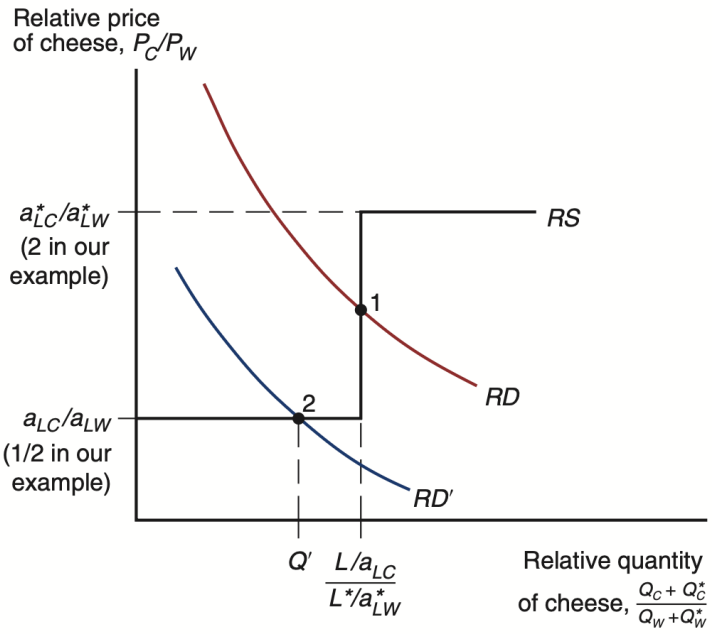


Figure 4: World Relative Supply and Relative Demand

Note that despite home only *partially specialize* in cheese at Point 2, it remains true that **a country will always specialize in the industry where it has a comparative advantage**.

2.3 Gains from Trade: The "Indirect Production" Method

Trade allows consumption possibilities to expand beyond the production possibilities frontier (PPF).

1. **Scenario A: Direct Production (Autarky)**
   Workers use the hour to produce Wine directly.

$$\text{Wine Obtained} = \frac{1}{a_{LW}}$$

2. **Scenario B: Indirect Production (Trade)**
   Workers use the hour to produce Cheese ($1/a_{LC}$), then trade it for Wine at the relative world price (P_C/P_W).

$$\text{Wine Obtained} = \underbrace{\left(\frac{1}{a_{LC}}\right)}_{\text{Cheese Made}} \times \underbrace{\left(\frac{P_C}{P_W}\right)}_{\text{Exchange Rate}}$$

Because Home has a comparative advantage in Cheese, the relative world price of Cheese is higher than Home's opportunity cost ($\frac{P_C}{P_W} > \frac{a_{LC}}{a_{LW}}$). Therefore, Indirect Production yields more wine:

$$\frac{1}{a_{LC}} \cdot \frac{P_C}{P_W} > \frac{1}{a_{LW}}$$

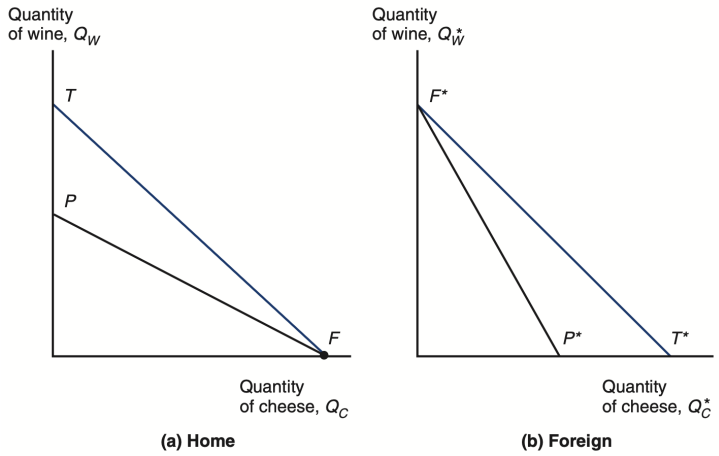


Figure 5: Gains from Trade: Expansion of Consumption Possibilities

2.4 Relative Wages

Relative wages are determined by relative productivity and relative prices. If Home specializes in Cheese and Foreign in Wine:

$$\text{Home Wage: } w = \frac{P_C}{a_{LC}}; \quad \text{Foreign Wage: } w^* = \frac{P_W}{a^*_{LW}}$$

$$\text{Relative Wage: } \frac{w}{w^*} = \frac{P_C}{P_W} \cdot \frac{a^*_{LW}}{a_{LC}}$$

The relative wage lies between the ratio of productivities in the two industries:

$$\frac{a^*_{LW}}{a_{LW}} \leq \frac{w}{w^*} \leq \frac{a^*_{LC}}{a_{LC}}$$

This confirms that a country can have high wages (due to absolute advantage/high productivity) and still trade beneficially based on comparative advantage.

2.5 Common Misconceptions

- **Productivity:** Trade benefits do **not** depend on absolute advantage. An unproductive country still gains by importing goods it produces inefficiently.
- **Pauper Labor Argument:** Trade with low-wage countries does not hurt high-wage countries; wages reflect productivity.
- **Exploitation:** Trade does not exploit low-wage workers if the alternative (autarky) leaves them worse off.

2.6 Ricardian Model with Many Goods

Suppose there are N goods. We rank them by Home's relative productivity (lowest relative unit labor requirement):

$$\frac{a_{L1}}{a^*_{L1}} < \frac{a_{L2}}{a^*_{L2}} < \cdots < \frac{a_{LN}}{a^*_{LN}}$$

Rule of Allocation: A good i is produced at Home if Home's unit labor cost is lower than Foreign's:

$$w \cdot a_{Li} < w^* \cdot a^*_{Li} \Rightarrow \frac{a^*_{Li}}{a_{Li}} > \frac{w}{w^*}$$

- Goods to the left of the relative wage threshold (w/w^*) are produced at Home.
- Goods to the right are produced abroad.

Determination of Relative Wage (w/w^*): Determined by the intersection of Relative Demand for Labor (derived from demand for goods) and Relative Supply of Labor (L/L^*).

- As w/w^* rises, Home goods become relatively more expensive, reducing demand for Home labor (downward sloping RD for labor).

2.7 Empirical Evidence (Balassa 1963)

The model predicts that countries export goods where their labor productivity is relatively high.

- Data shows a strong positive correlation between the ratio of U.S./UK exports and the ratio of U.S./UK labor productivity.
- Even if the U.S. has an absolute advantage in all sectors, trade follows the comparative advantage pattern.

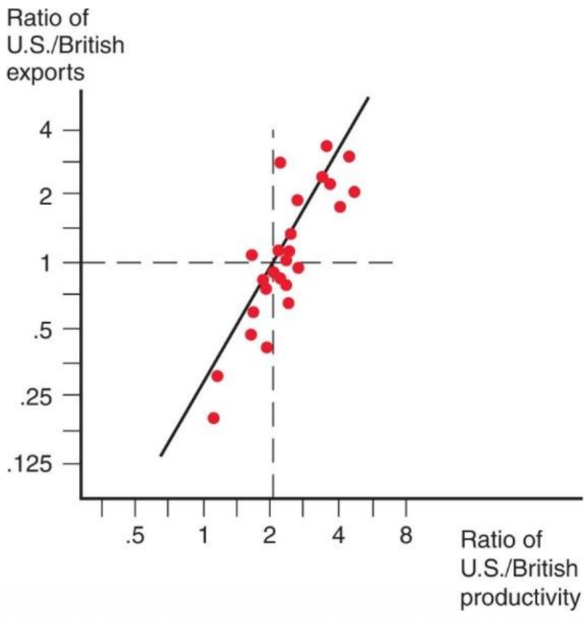


Figure 6: Productivity and Exports: A Test of the Ricardian Model

Specific Factors and Income Distribution

Ambrose W

1 The Specific Factors Model

The Specific Factors Model, developed by Paul Samuelson and Ronald Jones, analyzes the distributional effects of trade in the ***short run*** when some factors of production are immobile between sectors.

1.1 Assumptions

1. **Three Factors of Production:**
   - Labor (L)
   - Land (T)
   - Capital (K)
2. **Two Goods:**
   - **Cloth (C):** Produced using Capital (K) and Labor (L), but not Land.
   - **Food (F):** Produced using Land (T) and Labor (L), but not Capital.
3. **Factor Mobility:**
   - Labor is the **mobile factor** (can move between sectors).
   - Land and Capital are **specific factors** (can be used only in their respective sectors).
4. **Perfect Competition** prevails in all markets.

1.2 Production Functions

1.2.1 Cloth Sector

The production function for cloth is given by:

$$Q_C = Q_C(K, L_C)$$

where Q_C is the output of cloth and L_C is the labor input. The function is concave, subject to the **Diminishing Marginal Product of Labor**.

The distribution of gains in the cloth sector is derived as follows:

- **Labor Income in terms of Cloth:** $(\frac{w}{P_C}) \times L_C$
- **Capital Owner's Income (Real):** Total Output - Labor Income

Graphically, total output is the area under the MPL curve up to the labor input L_C.

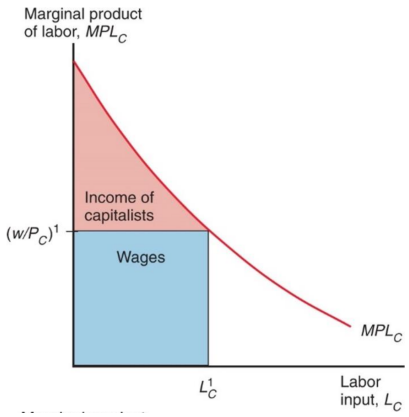


Figure 1: MPL & Distribution of Gains in the Cloth Sector

1.2.2 Food Sector

The production function for food is given by:

$$Q_F = Q_F(T, L_F)$$

Similar to cloth, this function exhibits diminishing marginal returns to labor.

1.2.3 Labor Constraint

The economy-wide labor force constraint is:

$$L_F + L_C = L$$

2 The Production Possibility Frontier (PPF)

The PPF is derived using a four-quadrant diagram that maps the allocation of labor to the production of both goods.

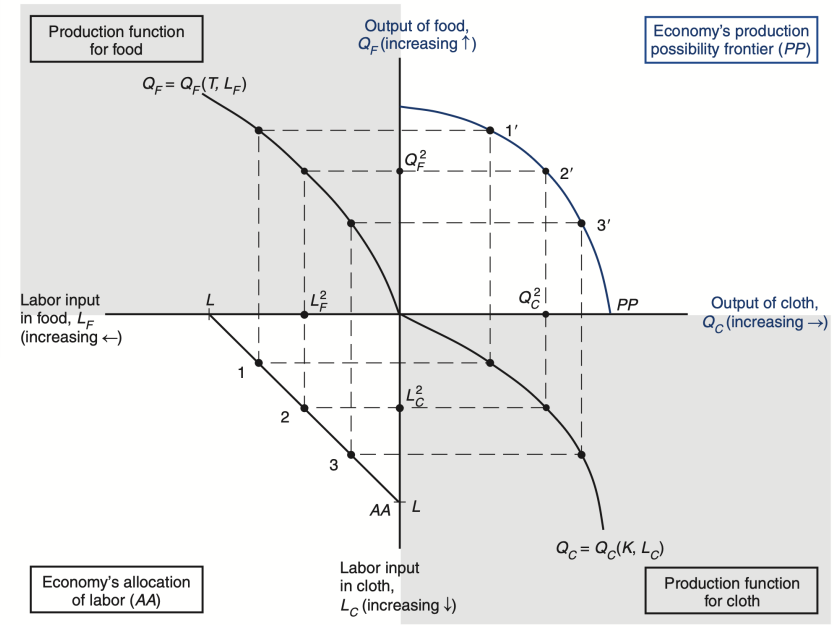


Figure 2: Derivation of the Production Possibility Frontier

2.1 Properties of the PPF

- **Curvature:** The PPF is curved (concave to the origin), reflecting diminishing marginal returns to labor in each industry.
- **Opportunity Cost:** The opportunity cost of cloth in terms of food represents the number of food units sacrificed for one extra unit of cloth. This is defined by the slope of the PPF:

$$Slope = -\frac{MPL_F}{MPL_C}$$

- **Interpretation:**
  - $\frac{1}{MPL_C}$: Number of labor hours required for one extra unit of cloth.
  - MPL_F: Quantity of food forgone to free up 1 hour of labor.

3 Prices, Wages, and Labor Allocation

3.1 Demand for Labor

Profit-maximizing employers hire workers up to the point where the value of the marginal product equals the wage rate:

$$MPL_C \times P_C = w$$
$$MPL_F \times P_F = w$$

Since labor is perfectly mobile, the wage rate (w) must be equalized across both sectors in equilibrium. If wages were unequal, labor would flow from the low-wage sector to the high-wage sector until equalization occurred.

3.2 Equilibrium Allocation

The labor market equilibrium is found at the intersection of the labor demand curves for the two sectors.

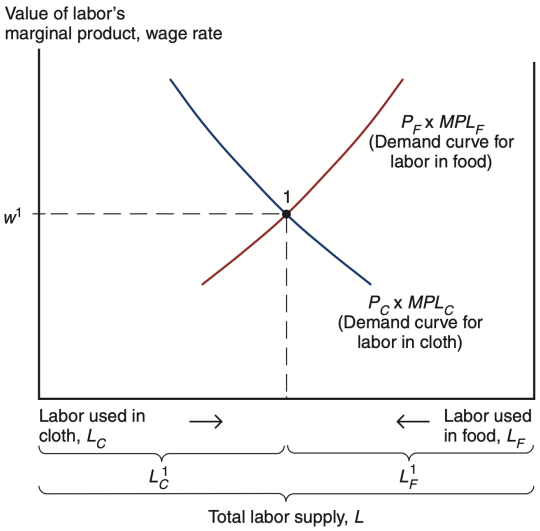


Figure 3: Sectoral Allocation of Labor

Mathematically, equilibrium requires:

$$MPL_C \times P_C = MPL_F \times P_F$$

This implies that at the optimal production point, the slope of the PPF equals the relative price ratio:

$$\textit{Slope of } PPF = -\frac{MPL_F}{MPL_C} = -\frac{P_C}{P_F}$$

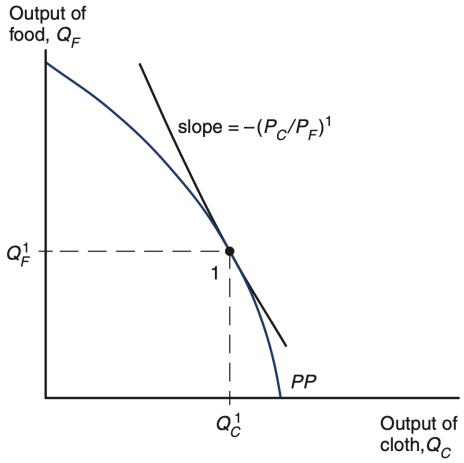


Figure 4: Optimal Production

4 Effects of Price Changes

4.1 Case 1: Equal-Proportional Change in Prices

Suppose both P_C and P_F increase by the same proportion (e.g., 10%).

- The labor demand curves in both sectors shift up by the same percentage.
- **Result:** The allocation of labor (L_C and L_F) remains **unchanged**.
- **Wages:** The nominal wage rate (w) rises by the same proportion as prices.
- **Real Variables:** Real wages ($\frac{w}{P_C}$, $\frac{w}{P_F}$) and the real incomes of capital and land owners remain unchanged.

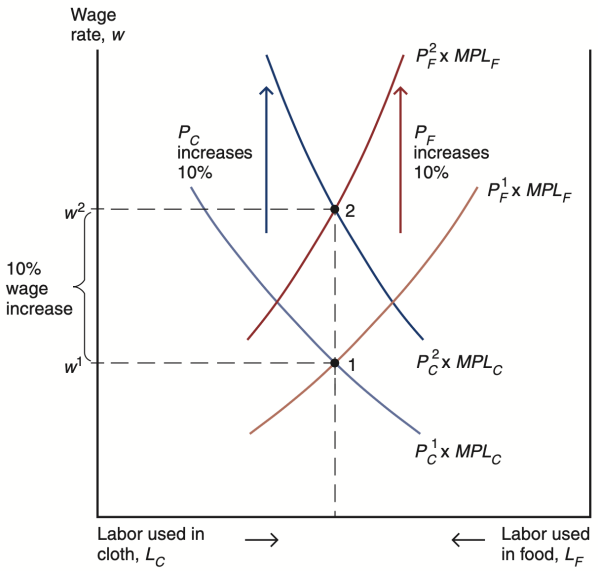


Figure 5: Allocation of Labor: Equi-Proportional Price Change

4.2 Case 2: Change in Relative Prices

Suppose the price of cloth increases ($P_C \uparrow$) by 7% while P_F remains constant.

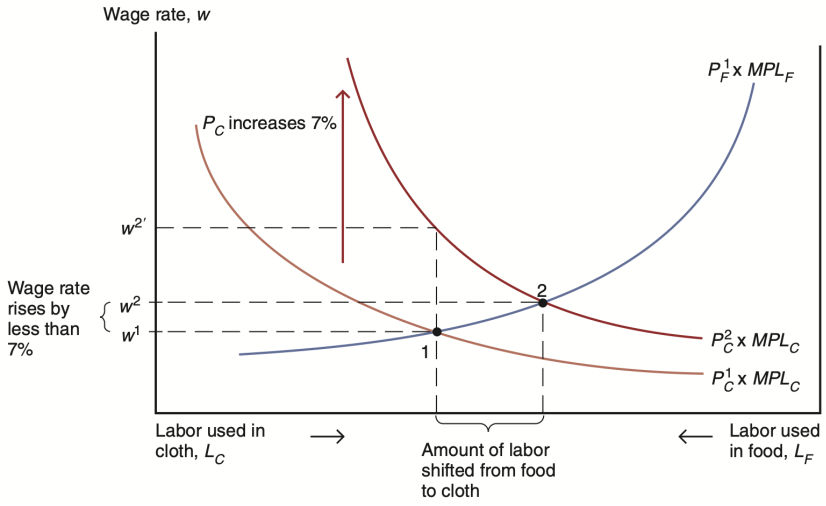


Figure 6: Allocation of Labor: Relative Price Change

4.2.1 Mechanism

1. The demand curve for labor in the cloth sector shifts upward ($P_C \times MPL_C$).
2. Labor shifts from the food sector to the cloth sector ($L_F \downarrow, L_C \uparrow$).
3. As L_C increases, MPL_C decreases; as L_F decreases, MPL_F increases.
4. The equilibrium wage rate (w) rises, but by **less** than the percentage increase in P_C.

4.2.2 Output Response

The economy moves along the PPF to a point with higher cloth output and lower food output.

$$\left(\frac{P_C^1}{P_F^1}\right) < \left(\frac{P_C^2}{P_F^2}\right) \Rightarrow \text{Slope of PPF becomes more negative} \Rightarrow Q_F \downarrow, Q_C \uparrow$$

4.2.3 Relative Supply Curve

The relationship between relative price and relative quantity is derived as the Relative Supply (RS) curve, which is upward sloping.

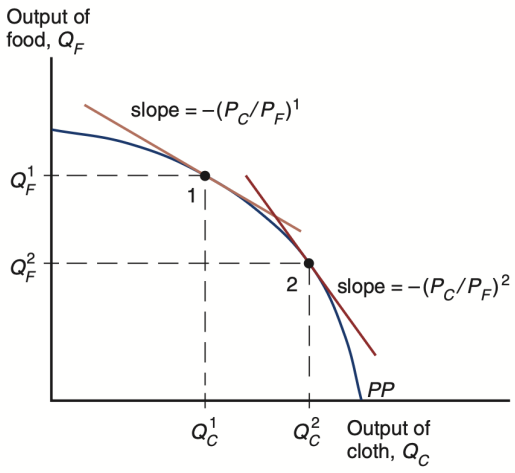


Figure 7: PPF: Relative Price Change

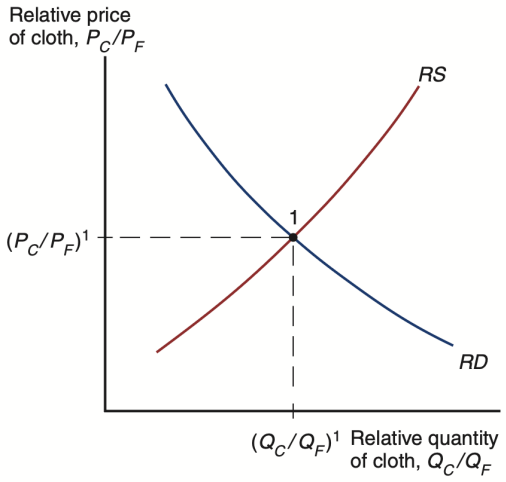


Figure 8: Relative Supply Curve

5 Distribution of Income

The welfare effects of a relative price increase in cloth ($P_C \uparrow$) are as follows:

5.1 Mobile Factor: Labor (Uncertain Welfare Effect)

The wage rate w rises, but less than proportionately to P_C.

- Real wage in terms of cloth falls: $\frac{w}{P_C} \downarrow$
- Real wage in terms of food rises: $\frac{w}{P_F} \uparrow$ (since $w \uparrow$ and P_F is constant)

Conclusion: The effect on workers is ambiguous and depends on the relative importance of cloth and food in their consumption preferences.

5.2 Specific Factor: Capital (Better Off)

Capital is specific to the sector whose relative price increased (Cloth).

$$P_C \uparrow \Rightarrow \left(\frac{w}{P_C}\right) \downarrow$$

Since the real wage in terms of cloth falls, the residual income left for capital owners increases. Additionally, the purchasing power of their income rises relative to food.
Conclusion: Owners of the specific factor in the expanding sector are strictly better off.

5.3 Specific Factor: Land (Worse Off)

Land is specific to the sector whose relative price decreased (Food).

$$w \uparrow \Rightarrow \left(\frac{w}{P_F}\right) \uparrow$$

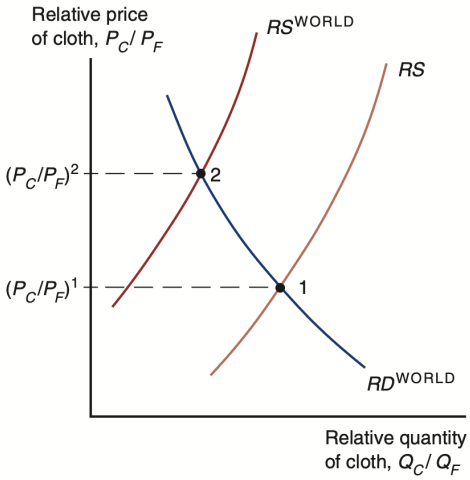
The real wage in terms of food rises, squeezing the income of landowners. Furthermore, the rising price of cloth reduces their purchasing power.
Conclusion: Owners of the specific factor in the contracting sector are strictly worse off.

6 International Trade in the Specific Factors Model

6.1 Trade Patterns and Relative Prices

Opening to trade changes the relative prices from the domestic autarky ratio to the world ratio (RS^{WORLD}).

- If the domestic relative price of cloth is lower than the world relative price, the country will export cloth.
- **Rule:** An economy exports the good whose relative price has increased and imports the good whose relative price has decreased.



6.2 Gains from Trade

Trade decouples production from consumption. The economy produces at the point where the PPF is tangent to the world price line, but consumers can choose a bundle along the budget constraint.

6.2.1 Budget Constraint

The budget constraint equates the value of consumption to the value of production:

$$P_C D_C + P_F D_F = P_C Q_C + P_F Q_F$$

Rearranging gives the import/export relationship:

$$D_F - Q_F = (P_C/P_F) \times (Q_C - D_C)$$

Where $(D_F - Q_F)$ is Food imports and $(Q_C - D_C)$ is Cloth exports. The slope of the budget constraint is $-(P_C/P_F)$.

Note that the autarky bundle (Point 1) is strictly dominated by the purple region. Unless Point 1 = Point 2, then there is always a part of the BC that allows simultaneous higher consumption of both goods

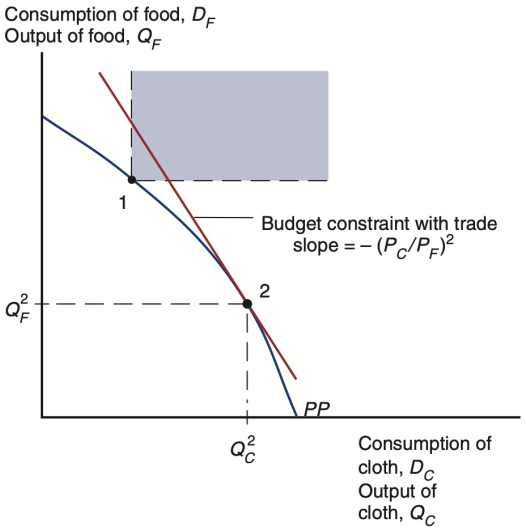


Figure 9: Gains from Trade: Production vs. Consumption

Conclusion on Welfare: Trade benefits the factor specific to the export sector while hurting the factor specific to the import-competing sectors; Effect of trade on workers are ambiguous.

7 Political Economy and Empirical Evidence

7.1 Why Protectionism Persists

Despite aggregate gains, protectionism is widespread due to:

1. **Inequality & Fairness:** Import-competing sectors often employ low-wage, unskilled workers who are hit hardest.

2. **Collective Action:** Beneficiaries of trade (consumers) are dispersed and disorganized, while losers (specific industries) are concentrated and well-organized (e.g., US sugar producers).

7.2 Trade and Unemployment

- **Sectoral Reallocation:** Trade shifts jobs from import-competing sectors to export sectors. This process is not instantaneous, causing transitional unemployment.

- **Data:** There is no evidence suggesting a positive correlation between unemployment rates and the import share of GDP over the long term. Unemployment is primarily a cyclical macroeconomic phenomenon.

7.3 The "China Shock"

While aggregate manufacturing output in the US has not declined (due to productivity), employment has. Autor, Dorn, and Hanson (2013) modeled the local labor market effects of import competition:

$$\Delta L_{it}^{m} = \gamma_t + \beta_1 \Delta IPW_{uit} + \beta X_{it}' + u_{it}$$

Where:

- ΔL_{it}^{m}: Growth of manufacturing employment in region i.
- ΔIPW_{uit}: Import exposure in local county i (instrumented by import patterns to other high-income countries).
- X_{it}: Control variables (demographics, routine-task share, etc.).

Finding: Import competition from China had sizable, adverse effects on US manufacturing employment in exposed local labor markets, with limited offsetting migration.

The Heckscher-Ohlin Model

Ambrose W

1 Introduction: Reasons for Trade

The Heckscher-Ohlin (HO) Model, also known as the Factor-Proportions Theory, posits that trade occurs due to differences in **factor endowments** across countries and differences in **factor intensities** across sectors.

- **Factor endowments (Country-level):** Countries differ significantly in their relative supplies of factors of production (Labor, Capital, Land, etc.).
- **Factor intensities (Sector-level):** Technologies differ across sectors. For example, the textile sector is more labor-intensive, while agriculture might be more land-intensive.

2 The Two-Factor Heckscher-Ohlin Model

2.1 Assumptions

1. **2x2x2 Structure:** Two Countries (Home & Foreign), Two Goods (Cloth C & Food F), Two Factors (Labor L & Capital K).
2. **Factor Supply:** Supply of L and K are fixed in each country, but vary *across* countries.
3. **Factor Mobility:** Both labor and capital are perfectly mobile across sectors in the long run. Consequently, the wage (w) and rental rate of capital (r) equalize across sectors ($w = r$).
4. **Technology & Preferences:** Same technology and same homothetic preferences across countries.

2.2 Production Possibilities

2.2.1 Production Functions and Unit Factor Requirements

The production functions are given by

$$Q_C = Q_C(K_C, L_C)$$

and

$$Q_F = Q_F(K_F, L_F)$$

We define the production technologies using unit factor requirements (which are **not** fixed in this model due to substitution): a_{xy} is the amount of factor x required to produce 1 unit of good y.

2.2.2 PPF without Factor Substitution (Example)

Suppose the following values: $a_{KC} = 2, a_{LC} = 2, a_{KF} = 3, a_{LF} = 1$; Total Capital $\omega_K = 3000$, Total Labor $\omega_L = 2000$.

The resource constraints are:

1. **Capital Constraint:** $a_{KC} \times Q_C + a_{KF} \times Q_F \leq \omega_K$
   Substituting values: $2Q_C + 3Q_F \leq 3000 \Rightarrow$ Slope $= -2/3$
2. **Labor Constraint:** $a_{LC} \times Q_C + a_{LF} \times Q_F \leq \omega_L$ Substituting values: $2Q_C + Q_F \leq 2000 \Rightarrow Slope = -\frac{a_{LC}}{a_{LF}} = -2$

The Production Possibility Frontier (PPF) is the interior boundary formed by these two constraints.

2.2.3 PPF with Factor Substitution

When factor substitution is allowed, the unit factor requirements are no longer fixed. The PPF becomes a smooth curve. The opportunity cost of cloth in terms of food increases as Q_C increases (concave PPF).

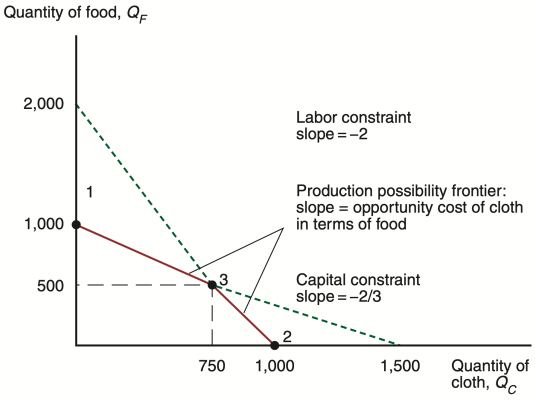


Figure 1: PPF w/o Factor Substitution

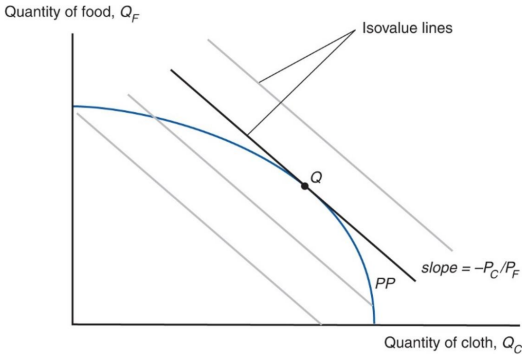


Figure 2: PPF with Factor Substitution.

2.3 Optimal Output Mix

The economy produces at the point that maximizes the value of production (V) given prices (See figure 2).

- **Isovalue Line:** Lines representing a constant value of production ($V = P_C Q_C + P_F Q_F$).
- The slope of the isovalue line is $Slope = -\frac{P_C}{P_F}$.
- **Equilibrium:** The economy produces at point Q where the PPF is tangent to the highest possible isovalue line. At Q, the relative price of cloth equals the slope of the PPF (opportunity cost).

3 Choosing Input Mix: Cost Minimization

3.1 Isoquants and Isocost Lines

Producers minimize cost by choosing the optimal ratio of Labor to Capital.

- **Unit Isoquant:** Combinations of K and L producing 1 unit of output (e.g., Food).

- **Isocost Line:** Combinations of L and K with the same total cost. $ra_{KF} + wa_{LF} = c$.
- The slope of the isocost line is $Slope = -(w/r)$.

The producer chooses the point on the unit isoquant tangent to the lowest possible isocost line.

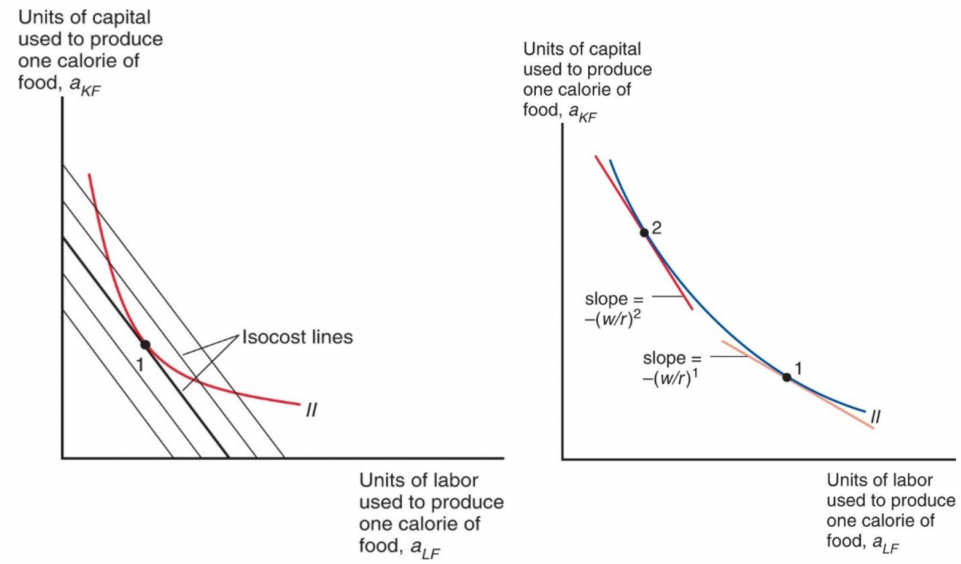


Figure 3: Optimal Input Mix: Tangency determines the optimal L/K ratio.

3.2 Factor Substitution Effect

When the wage-rental ratio (w/r) increases (labor becomes relatively more expensive):

- Isocost lines become steeper $(Slope = -(w/r))$.
- Producers substitute capital for labor.
- Result: $\frac{w}{r} \uparrow \Rightarrow \frac{L}{K} \downarrow \Rightarrow$ Downward sloping factor demand curve.

4 Relative Factor Demand and Intensity

4.1 Factor Intensity Definition

We assume Cloth is **labor-intensive** and Food is **capital-intensive**. This means that at *any* given wage-rental ratio (w/r), cloth uses more labor relative to capital than food does:

$$\frac{a_{LC}}{a_{KC}} > \frac{a_{LF}}{a_{KF}} \Rightarrow \frac{L_C}{K_C} > \frac{L_F}{K_F}$$

Note: Factor intensity depends on the L/K ratio used in production, not the ratio of factors to output.

4.2 Relative Factor Demand Curve

There is a downward-sloping relationship between the relative price of factors (w/r) and the relative quantity of factors used (L/K).

- FF: Factor demand for Food production.

- CC: Factor demand for Cloth production.
- Because Cloth is labor-intensive, the CC curve lies to the right of the FF curve.

Assuming no factor-intensity reversal:

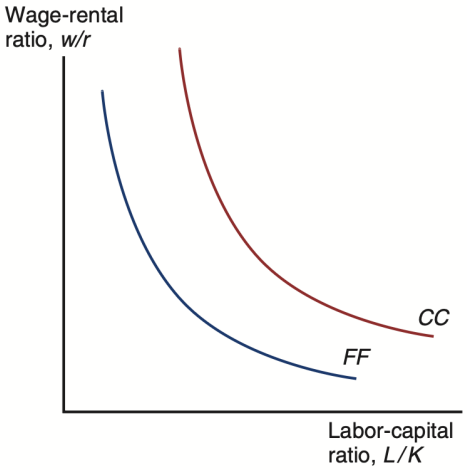


Figure 4: Relative Factor Demand Curves (CC and FF).

5 Factor Prices and Goods Prices

5.1 Unit Value Isoquants and Zero Profit

In a perfectly competitive market, firms earn zero profit. The cost to produce \$1 worth of a good must equal \$1.

$$P_F = c_F(w, r) = 1$$

$$P_C = c_C(w, r) = 1$$

Both sectors must share the same factor costs (w, r) due to perfect mobility.

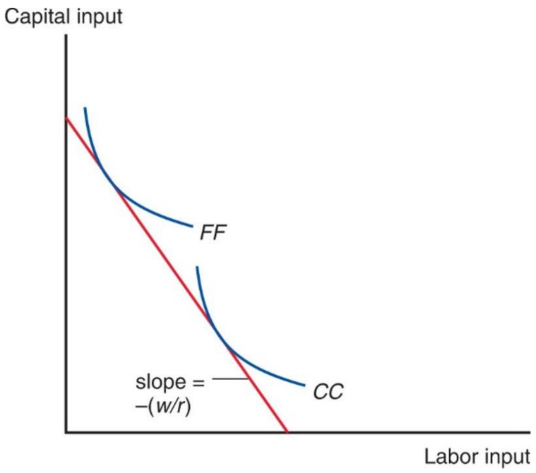


Figure 5: Factor Price Determination

Figure 6: Relationship between (P_C/P_F) and (w/r).

When price of output changes, the unit value isoquants shift, leading to a new equilibrium in factor prices. (e.g. $P_C \uparrow$ shifts CC to the left, raising w/r).

5.2 Stolper-Samuelson Theorem

Theorem: A rise in the relative price of a good increases the real return to the factor used intensively in that good and lowers the real return to the other factor.

Logic (The Magnification Effect): Suppose the relative price of cloth rises ($\frac{P_C}{P_F} \uparrow$).

1. Because Cloth is labor-intensive, the relative cost of labor must rise: $\frac{w}{r} \uparrow$.
2. As labor becomes relatively expensive, producers in **both** sectors switch to using less labor relative to capital: $\frac{L}{K} \downarrow$ in both industries.
3. **Marginal Product and Real Returns:**
   - Since $\frac{L}{K} \downarrow$, each worker has more capital to work with. Marginal Product of Labor (MPL) rises. Since $w = MPL \times P$, the real wage rises in terms of both goods.
   - Since $\frac{K}{L} \uparrow$ (or less labor per unit of capital), Marginal Product of Capital (MPK) falls. Real rental rates fall.

Distributional Effect: Increase in $\frac{P_C}{P_F}$ will unambiguously raise the purchasing power of workers and lower purchasing power of capital owners by raising real wages and lowering real rents in terms of BOTH goods.

$$\frac{P_C}{P_F} \uparrow \Rightarrow \text{Raise income of workers relative to capital owners}$$

This raises the real income of workers and lowers the real income of capital owners.

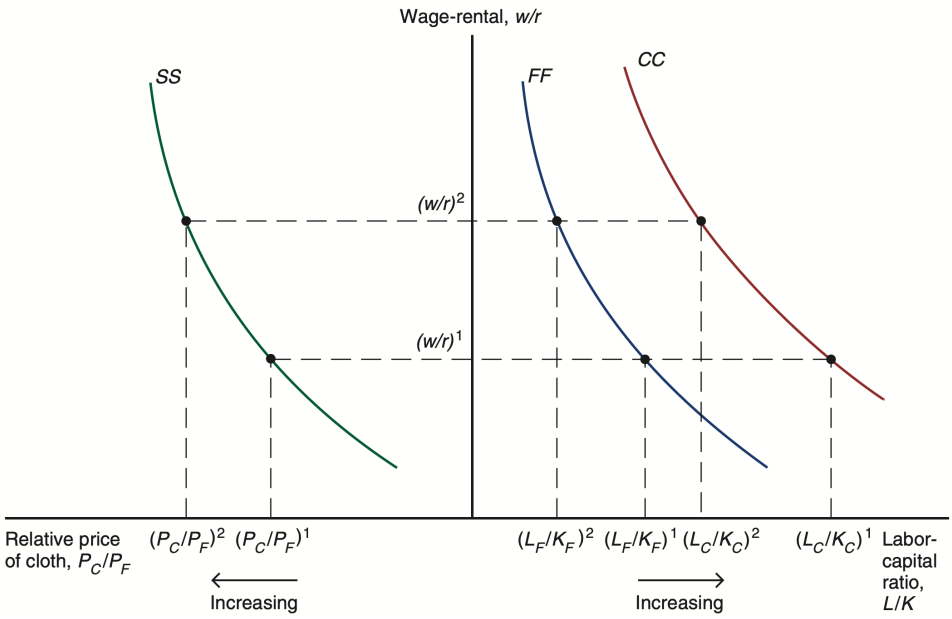


Figure 7: Stolper-Samuelson Effect

6 Resources and Output: Rybczynski Theorem

6.1 Effect of Endowment Change

Suppose the labor force increases ($L \uparrow$), increasing the economy's aggregate L/K ratio.

- At constant relative goods prices (P_C/P_F), the factor price ratio (w/r) and factor usage ratios (L/K) in each sector remain constant.
- To absorb the extra labor while maintaining constant L/K ratios in production, the economy must reallocate resources towards the labor-intensive sector (Cloth).
- Result: The economy produces more Cloth and less Food ($Q_C \uparrow, Q_F \downarrow$).

6.2 Rybczynski Theorem

Holding output prices constant, as the supply of a Factor of Production increases, the supply of the good that uses this factor intensively increases, and the supply of the other good decreases. This causes a **biased expansion** of the PPF.

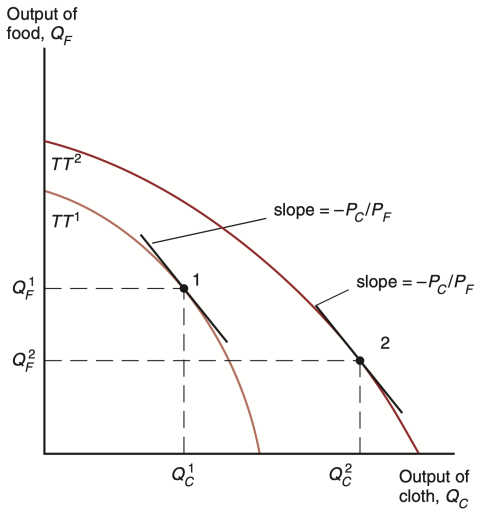


Figure 8: Rybczynski Effect: Biased expansion of the PPF due to an increase in Labor.

7 International Trade and the Heckscher-Ohlin Theorem

7.1 Trade Equilibrium

- **Assumption:** Home is *relatively* labor-abundant; Foreign is relatively capital-abundant.

$$\frac{L}{K} > \frac{L^*}{K^*}$$

- **Relative Supply:** Home will be relatively efficient at producing Cloth (labor-intensive). Home's Relative Supply (RS) curve of cloth lies to the right of Foreign's (RS^*).
- **Trade Pattern:** Trade leads to price convergence. The world relative price lies between the autarky prices:

$$\left(\frac{P_C}{P_F}\right)^1 < \left(\frac{P_C}{P_F}\right)^2 < \left(\frac{P_C}{P_F}\right)^3$$

where 1 is Home Autarky, 2 is World, 3 is Foreign Autarky.

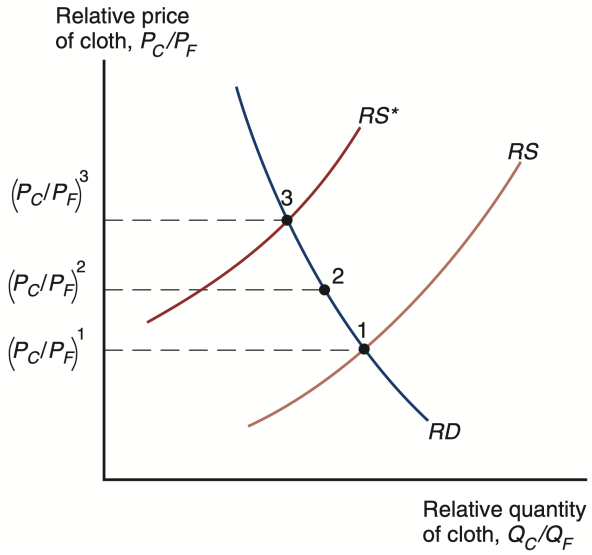


Figure 9: International Trade Equilibrium: Convergence of Relative Prices.

7.2 Heckscher-Ohlin Theorem

Definition: The country that is abundant in a factor exports the good whose production is intensive in that factor.

- Home (Labor Abundant) exports Cloth (Labor Intensive).
- Foreign (Capital Abundant) exports Food (Capital Intensive).

7.3 Distribution of Income Implication

In the long-run,

1. Opening to trade **expands an economy's consumption possibilities**; Redistribution allows everyone to be better off.
2. **Owners of a country's abundant factors gain from trade**, but owners of a country's scarce resource lose.
3. Labor wins regardless of sector in which they actually employed

8 Discussions and Empirical Results

8.1 Trade and Income Inequality (North-South Trade)

According to the HO model and Stolper-Samuelson theorem, opening to trade in a skill-abundant country (like the US) should raise the premium for skilled labor and lower the return to unskilled labor.

$$\frac{P_H}{P_L} \uparrow \Rightarrow \frac{w_s}{w_u} \uparrow$$

However, empirical data shows that the ratio of skilled to unskilled employment has increased across *all* sectors, which contradicts the simple substitution effect of the HO model.

8.1.1 Alternative Explanation: Skill-Biased Technological Change

Technological change has increased the relative demand for skilled workers in all sectors.

- **Trade Explanation:** Would cause a shift *along* the demand curve (lower demand for skilled labor as its price rises).
- **Tech Change Explanation:** Shifts the relative demand curve (LL and HH) outward. Firms employ more skilled labor despite the rising wage premium ($w_s/w_u \uparrow$).

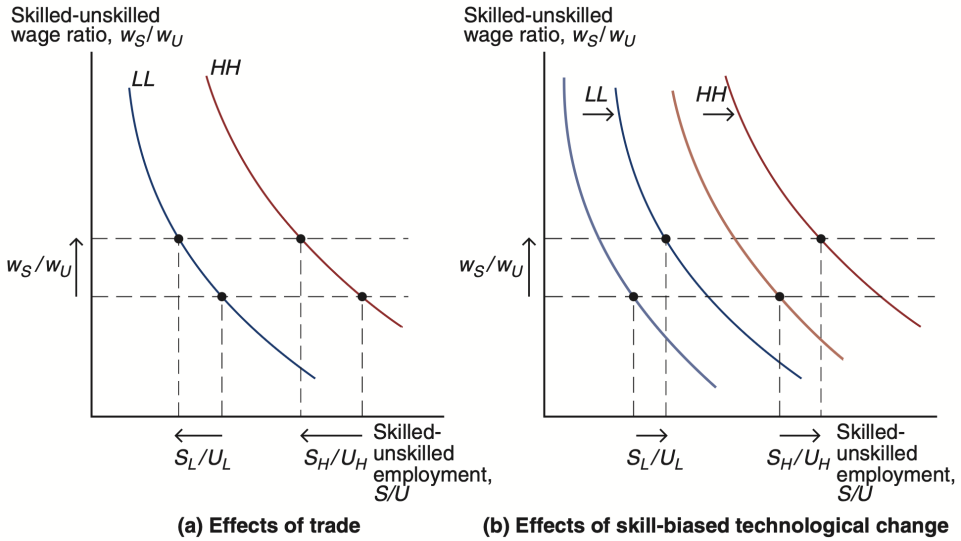


Figure 10: (a) Effect of Trade vs (b) Effect of Skill-Biased Technological Change on Wage Inequality.

8.2 Factor Price Equalization (FPE)

In theory, trade allows complete convergence in relative goods prices, which should lead to complete convergence in factor prices (wages and rents). However, in reality, wages are not equalized (e.g., US wages vs. Mexico wages). Reasons include:

1. Differences in technology (affecting productivity).
2. Trade costs (transportation, barriers) prevent goods price equalization.
3. Factor immobility in the short run.

8.3 The Leontief Paradox

Data from 1953 showed that U.S. exports were **less** capital-intensive than U.S. imports, despite the U.S. being the most capital-abundant country.

	Imports	**Exports**
Capital per million dollars	\$2,132,000	\$1,876,000
Labor (person-years) per million dollars	119	131
Capital-labor ratio	\$17,916	\$14,321

Table 1: The Leontief Paradox (1953 Data)

The Standard Trade Model

Ambrose W

1 Production Possibilities and Relative Supply

1.1 Production Possibilities Frontier (PPF)

We assume each country produces two goods: Food (F) and Cloth (C). We allow factor substitution, leading to a concave PPF.

1.2 Iso-value Lines and Optimal Production

Firms seek to maximize the market value of their output (V).

- **Iso-value Lines:** Combinations of quantities produced that yield the same market value.
- The equation of the iso-value line is:

$$V = P_C Q_C + P_F Q_F$$

- Rearranging for Q_F (to plot on the vertical axis):

$$Q_F = \frac{V}{P_F} - \left(\frac{P_C}{P_F}\right) Q_C$$

- **Optimal Production Choice:** The economy produces at point Q, where the PPF is tangent to the highest possible iso-value line. At this point, the slope of the PPF equals the negative relative price of cloth ($-P_C/P_F$).

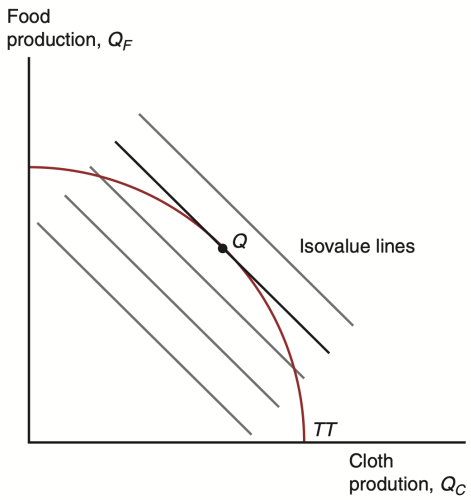


Figure 1: Production Possibilities and Optimal Production Choice (Q)

1.3 Relative Supply (RS) Curve

There is a positive relationship between the relative price of cloth and the relative production of cloth.

- When the relative price of cloth rises ($P_C/P_F \uparrow$), the isovalue lines become steeper.
- Production shifts: $Q_C \uparrow$ and $Q_F \downarrow$.
- Consequently, the relative supply curve (RS) of cloth slopes upward.

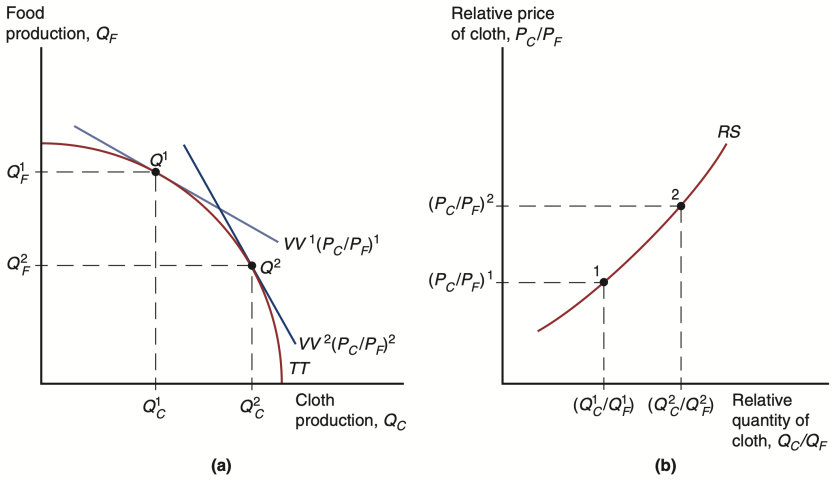


Figure 2: Relative Supply Curve of Cloth

2 Relative Prices and Demand

2.1 Optimal Consumption

The value of consumption must equal the value of production (assuming trade balance = 0).

$$P_C Q_C + P_F Q_F = P_C D_C + P_F D_F = V$$

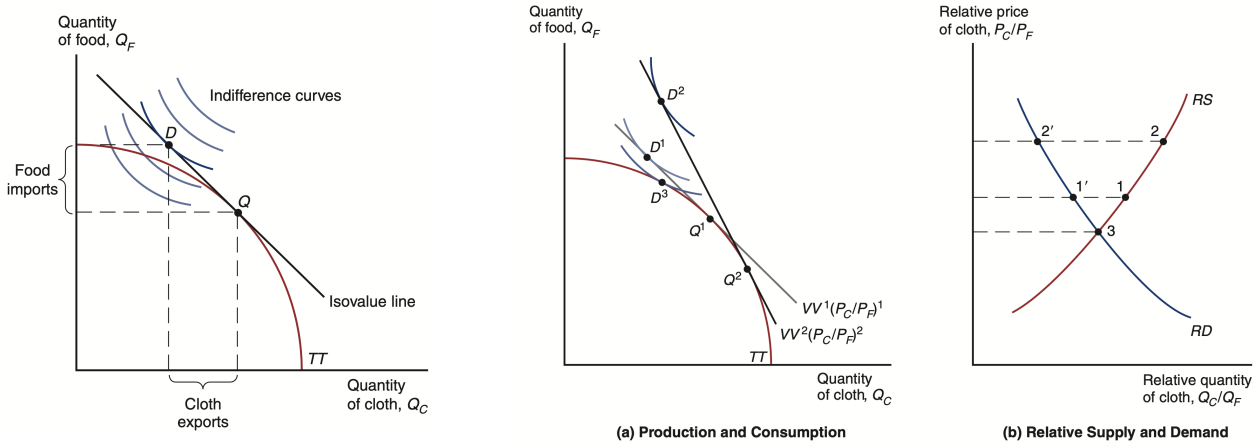
Therefore, production and consumption share the same iso-value line.

- **Optimal Choice:** Consumption occurs at Point D, where the highest indifference curve is tangent to the iso-value line.
- **Trade Pattern:**
  - If $Q_F < D_F$: Import Food.
  - If $Q_C > D_C$: Export Cloth.
  - Autarky: Point D^3 (IC tangent to PPF).

2.2 Relative Demand (RD) Curve

We analyze the effect of an increase in the relative price of cloth ($\frac{P_C}{P_F} \uparrow$):

1. **Income Effect:** Since the price of the economy's exporting commodity increases, the purchasing power of the economy increases. Consumption of both goods typically increases ($D_F \uparrow$ and $D_C \uparrow$).



2. **Substitution Effect:** Cloth becomes relatively more expensive. Consumers substitute away from cloth ($D_C \downarrow$ and $D_F \uparrow$).

Net Effect:

- Demand for Food (D_F) increases unambiguously.
- Demand for Cloth (D_C) effect is theoretically ambiguous, but we assume the substitution effect dominates.
- Therefore, relative demand for cloth (D_C/D_F) decreases as relative price rises. The RD curve slopes downward.

3 Welfare Effect of Changes in Terms of Trade

3.1 Definition

The Terms of Trade (ToT) is defined as the price of the good a country exports divided by the price of the good it imports.

$$TT = \frac{P_X}{P_M}$$

3.2 Welfare Rule

A rise in the ToT increases a country's welfare, while a decline in ToT reduces its welfare.
Important Note: Changes in ToT can **NEVER** decrease the country's welfare below its welfare level in the absence of trade (Autarky).

4 Determining Relative Prices (World Equilibrium)

4.1 Assumptions

- **Home:** Exporter of Cloth. $TT = \frac{P_C}{P_F}$.
- **Foreign:** Exporter of Food. $TT^* = \frac{P_F}{P_C}$.
- Trade patterns result from differences in production capabilities (Home is better endowed for Cloth; Foreign for Food).
- **Demand:** Both countries share the same preferences and same relative demand curve ($RD = RD^* = RD^{WORLD}$).

4.2 World Relative Supply (RS^{WORLD})

By construction, the World RS curve must lie between the Home RS and Foreign RS curves. It is calculated by summing quantities supplied:

$$RS^{WORLD} = \frac{Q_C + Q_C^*}{Q_F + Q_F^*}$$

Because Home is biased toward Cloth, $\frac{Q_C}{Q_F} > \frac{Q_C^*}{Q_F^*}$ at any given price.

4.3 Equilibrium

The equilibrium relative price $(\frac{P_C}{P_F})^1$ is found at the intersection of RS^{WORLD} and RD.

- Home's desired export of cloth = Foreign's desired import of cloth:

$$Q_C - D_C = D_C^* - Q_C^*$$

- Home's desired import of food = Foreign's desired export of food:

$$D_F - Q_F = Q_F^* - D_F^*$$

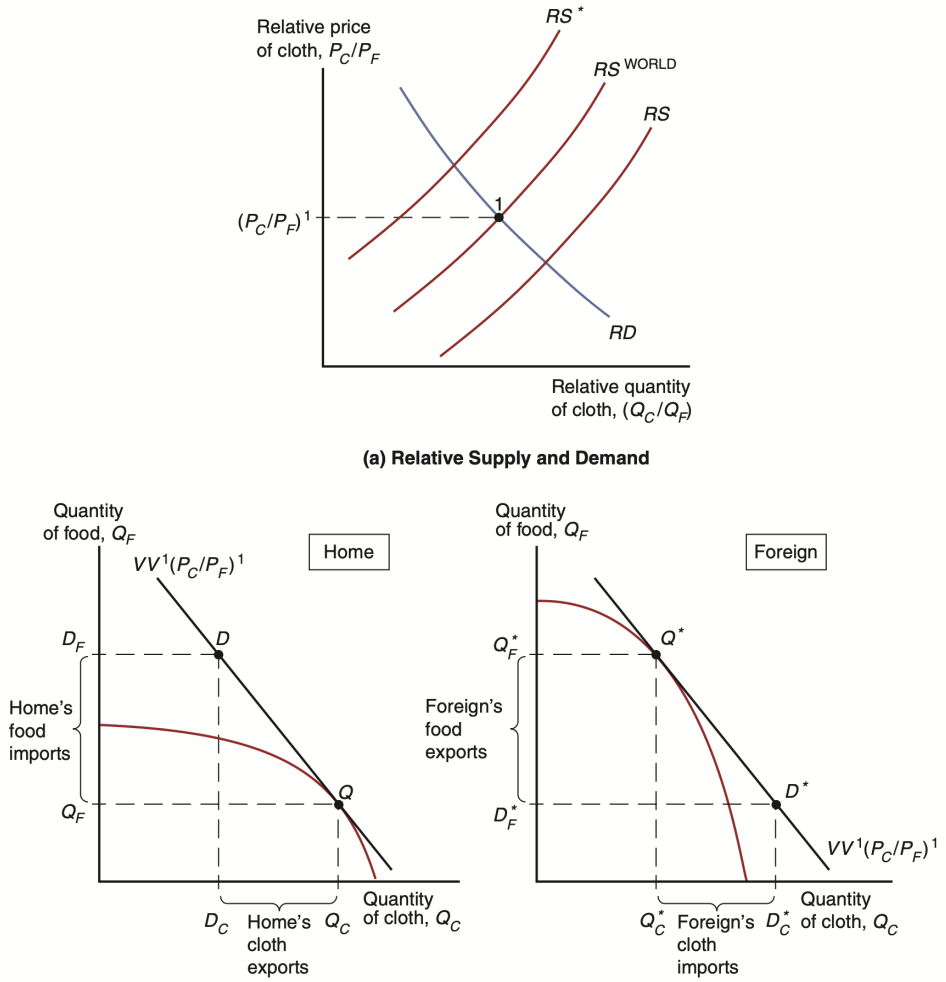


Figure 3: World General Equilibrium

5 Economic Growth and the PPF

5.1 Biased Growth

Growth is rarely uniform; it is usually biased, shifting the PPF out more in one direction.

- **Ricardian Source:** Technological advancement in one sector.
- **Heckscher-Ohlin Source:** Growth is biased in the direction of the good whose production is intensive in the factor whose supply has increased.

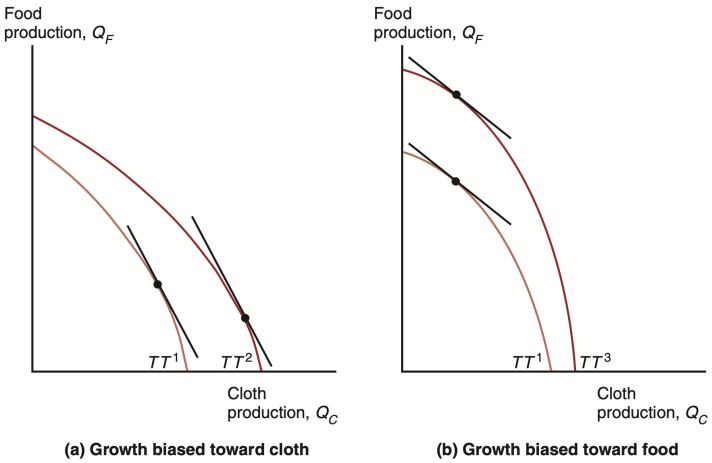


Figure 4: PPF: Baised Growth

6 World Relative Supply and Terms of Trade

We analyze how biased growth affects the World RS curve and subsequently the ToT.

6.1 Cloth-Biased Growth (Home)

If Home experiences growth biased toward cloth ($Q_C \uparrow, Q_F \downarrow$ at any given price):

- The World RS curve shifts to the right ($RS^1 \to RS^2$).
- **Formula:**
$$RS^2 = \frac{Q_C \uparrow + Q_C^*}{Q_F \downarrow + Q_F^*} > \frac{Q_C + Q_C^*}{Q_F + Q_F^*} = RS^1$$
- **Price Effect:** Relative price of cloth decreases: $(\frac{P_C}{P_F})^1 > (\frac{P_C}{P_F})^2$.
- **ToT Effect:** Worsening ToT for Home (Cloth exporter); Improving ToT for Foreign.
$$TT = \frac{P_C}{P_F} \downarrow \; TT^\star = \frac{P_F}{P_C} \uparrow$$

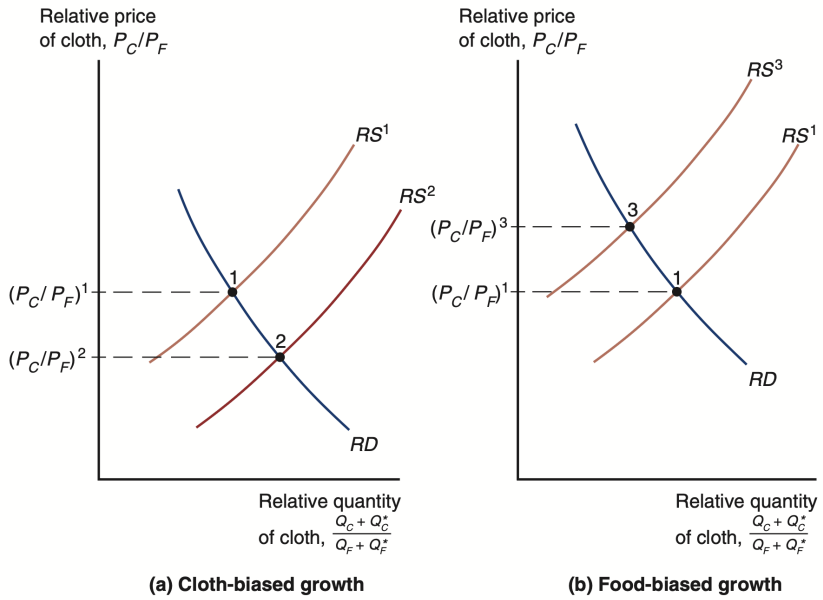


Figure 5: Relative Supply: Biased Growth

6.2 Food-Biased Growth

If growth is biased toward food:

- The World RS curve shifts to the left ($RS^1 \rightarrow RS^3$).
- **Price Effect:** Relative price of cloth increases. $\frac{P_C}{P_F} \uparrow$
- **ToT Effect:** Improving ToT for Home ($TT \uparrow$); Worsening ToT for Foreign ($TT^\star \downarrow$).

6.3 Summary of Growth Types

- **Export-biased growth:** Growth that disproportionately expands a country's PPF in the direction of the good it exports. Tends to **worsen** the growing country's ToT.
- **Import-biased growth:** Growth that disproportionately expands a country's PPF in the direction of the good it imports. Tends to **improve** the growing country's ToT.

7 International Effects of Growth (Welfare Analysis)

7.1 Welfare Effect of Export-Biased Growth (Home)

1. **Direct Effect:** Expanded PPF improves Home's welfare.
2. **Indirect Effect:** Worsening Home's ToT, partially offsetting the welfare improvement.

Net Result: Ambiguous for Home. Foreign welfare unambiguously increases.

7.2 Welfare Effect of Import-Biased Growth (Home)

1. **Direct Effect:** Expanded PPF improves Home's welfare.
2. **Indirect Effect:** Improving Home's ToT.

Net Result: Home welfare unambiguously increases. Foreign welfare falls.

8 Tariffs and Export Subsidies

Tariffs and subsidies create a "wedge" between internal (domestic) prices and external (world) prices.

- **Key Distinction:** ToT corresponds to *external* prices, not internal prices.

8.1 Import Tariff (Imposed by Home on Food)

Home imposes a tariff on its imported good (Food).

- **Internal Price:** The internal price of food (the imported good) rises relative to cloth.

$$\left(\frac{P_C}{P_F}\right)_{in} < \left(\frac{P_C}{P_F}\right)_{ex}$$

- **Supply Shift:** Home producers (who use internal price) face lower relative cloth prices $\Rightarrow Q_C \downarrow$ and $Q_F \uparrow$. World RS shifts to the **left**.
- **Demand Shift:** Home consumers face expensive food, shift consumption to cloth $\Rightarrow D_C \uparrow$ and $D_F \downarrow$. World RD shifts to the **right**.
- **Result:** World relative price of cloth $(\frac{P_C}{P_F})_{ex}$ rises.
- **ToT Effect:** Home's Terms of Trade improves ($P_C/P_F \uparrow$).
- **Welfare:** Ambiguous. If the country is large enough, the ToT gain may outweigh the distortion loss. If small, net loss.

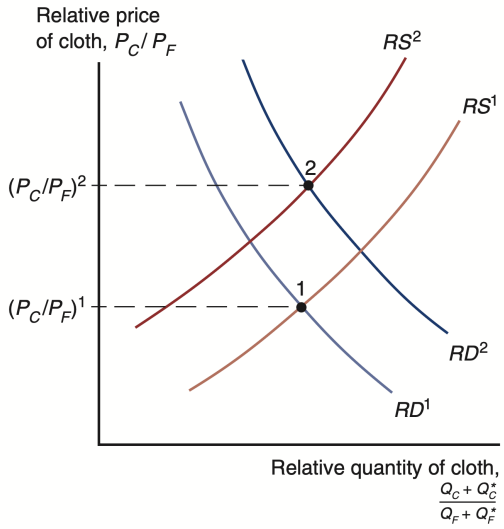


Figure 6: Simultaneous Shifts in RS and RD due to Import Tariff

8.2 Export Subsidy (Imposed by Home on Cloth)

Home provides a subsidy to cloth exporters.

- **Internal Price:** The internal price of cloth must rise (producers will not sell domestically unless paid the export price + subsidy).

$$\left(\frac{P_C}{P_F}\right)_{in} > \left(\frac{P_C}{P_F}\right)_{ex}$$

- **Supply Shift:** Higher internal relative price of cloth $\Rightarrow Q_C \uparrow$ and $Q_F \downarrow$. World RS shifts to the **right**.
- **Demand Shift:** Consumers shift from cloth to food $\Rightarrow D_C \downarrow$ and $D_F \uparrow$. World RD shifts to the **left**.
- **Result:** World relative price of cloth $(\frac{P_C}{P_F})_{ex}$ falls.
- **ToT Effect:** Home's Terms of Trade deteriorates.
- **Welfare:** Unambiguously negative. Home suffers from both ToT deterioration and distortion/fiscal costs.

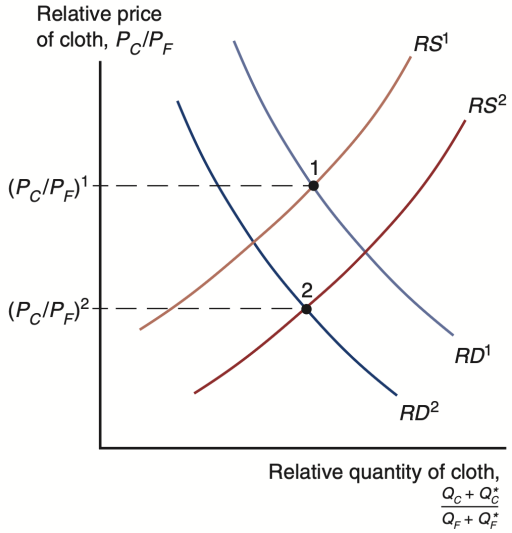


Figure 7: Simultaneous Shifts in RS and RD due to Export Subsidy

9 Intertemporal Borrowing and Lending

9.1 Concept

The Standard Trade Model can be modified to account for trade over time (Intertemporal Trade).

- **Goods:** Current consumption and Future consumption.
- **Intertemporal PPF:** Represents the trade-off between consuming today vs. tomorrow.

9.2 Relative Price of Future Consumption

If the real interest rate is r, 1 unit of current consumption is worth $(1 + r)$ units of future consumption.

- The price of future consumption relative to current consumption is:

$$\frac{1}{1+r}$$

9.3 Equilibrium

- **Home Bias:** If Home's PPF is biased toward current output, it exports current consumption (lends).
- **Foreign Bias:** If Foreign's PPF is biased toward future output, it exports future consumption (borrows/repays later).
- Equilibrium determines the world real interest rate.

External Economies of Scale & International Location of Production

Ambrose W

1 Economies of Scale and Market Structure

1.1 Limitations of Comparative Advantage Models

Old trade models (Ricardian, Heckscher-Ohlin, Specific Factors) rely on assumptions of **Constant Returns to Scale (CRS)**. While these models effectively explain *North-South trade* (trade between dissimilar countries based on productivity or factor endowment differences), they fail to adequately explain *North-North trade* (trade between similar countries).

1.2 Returns to Scale

Modern trade theory incorporates **Increasing Returns to Scale (IRS)**, where output increases at a rate disproportionately higher than the increase in inputs.

- **Internal Economies of Scale:** The total cost per unit of output depends on the size of the *individual firm*. This typically leads to an **imperfectly competitive** market structure (e.g., oligopoly, monopoly) because larger firms possess a distinct cost advantage over smaller ones.
- **External Economies of Scale (EOS):** The total cost per unit of output depends on the size of the *industry* within a specific region, not the size of the individual firm. This structure is compatible with **perfect competition**, as the industry is comprised of many small firms that benefit collectively from the industry's aggregate size.

2 External Economies of Scale

Alfred Marshall identified three principal drivers explaining why industrial clusters (e.g., Silicon Valley, Hollywood, specialized manufacturing towns in China) emerge:

2.1 Specialized Suppliers (Input Sharing)

A localized industrial cluster creates a market large enough to support specialized suppliers of equipment and services.

- Firms can outsource non-core activities to specialized, low-cost suppliers.
- **Example:** In the 1990s, Silicon Valley firms could contract out chip-making equipment and chemicals, avoiding the resource drain of vertical integration.

2.2 Labor Market Pooling

Clusters create a pooled market for workers with highly specialized skills.

- **Benefit to Firms:** Access to a steady supply of specific talent, reducing hiring costs.

- **Benefit to Workers:** Reduced risk of unemployment and lower job-search costs due to the density of potential employers.

2.3 Knowledge Spillovers

In innovation-heavy industries, the informal exchange of information is paramount.

- Physical proximity facilitates face-to-face interactions.
- Ideas flow between firms through social mixing and employee mobility, acting as a positive externality.

3 The Theory of External Economies

3.1 The Forward-Falling Supply Curve

Under external EOS, a larger industry output (Q) leads to lower average costs (AC) for all firms.

- We assume perfect competition; thus, price equals marginal cost ($P = MC = AC$).
- The industry supply curve is **forward-falling**: as industry demand and output increase, the price at which firms are willing to sell decreases.

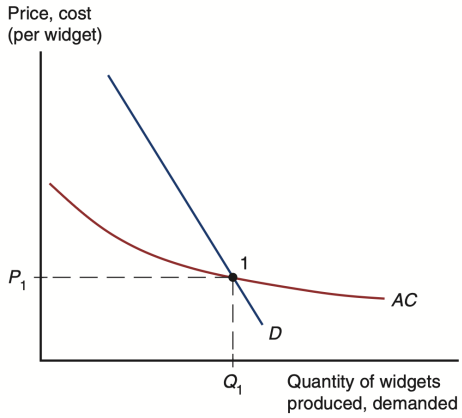


Figure 1: External Economies of Scale: Forward-Falling Supply Curve

4 International Trade and External EOS

4.1 Equilibrium Before Trade (Autarky)

Consider two countries, China and the US, producing buttons. Assume China has lower production costs at any given quantity (perhaps due to lower wages), such that $P_{CHINA} < P_{US}$.

4.2 Equilibrium with Free Trade

When trade opens:

1. The Chinese industry expands to supply the world market because $P_{CHINA} < P_{US}$.

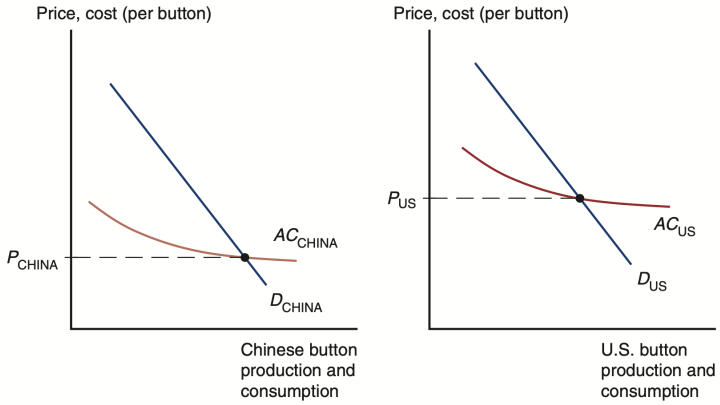


Figure 2: Autarky Equilibrium in China and the US

2. As Chinese production (Q) increases, their Average Cost (AC) falls further (due to External EOS).
3. The US industry contracts and eventually exits.
4. **Result:** Production concentrates entirely in China. The world price (P_2) falls below the pre-trade price of the most efficient producer (P_1).

$$P_2 < P_1 = P_{CHINA} < P_{US} \tag{1}$$

Unlike standard comparative advantage models which predict price convergence (intermediate prices), External EOS models predict a reduction in global prices everywhere.

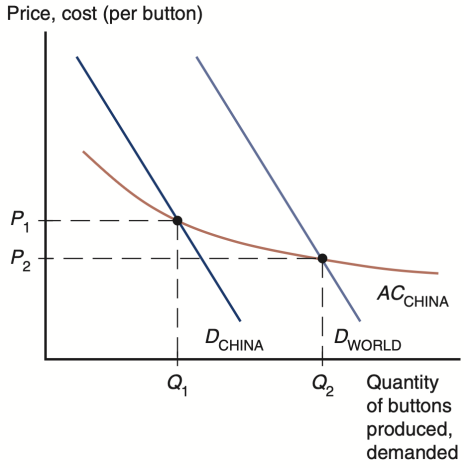


Figure 3: World Equilibrium with External Economies

5 Determinants of Location and "Lock-in" Effects

Why does a specific location become the global hub?

- **Comparative Advantage:** Underlying differences in resources or productivity.
- **Historical Contingency:** An initial advantage (often random) allows a location to establish critical mass first.

5.1 First-Mover Advantage

Consider China (incumbent) and Vietnam (potential entrant). Suppose Vietnam could theoretically produce buttons more cheaply ($AC_{VIETNAM} < AC_{CHINA}$) due to factor endowments.

- China is already producing at world scale Q_1 with price P_1.
- Vietnam has no production ($Q = 0$). The cost for the first unit is C_0.
- Because $C_0 > P_1$, no individual Vietnamese firm can enter the market profitably, despite the country's potential long-run efficiency.

This leads to **Lock-in**: Industries may remain in the "wrong" location due to established scale advantages.

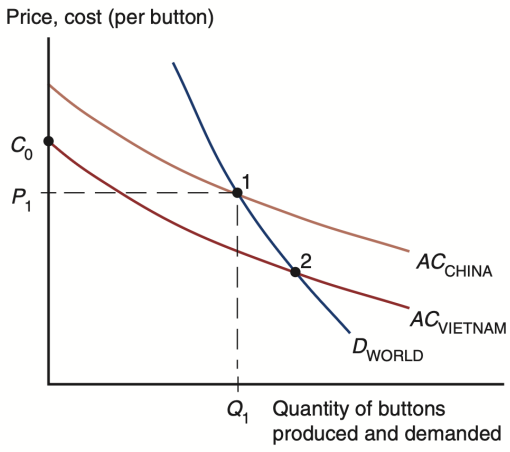


Figure 4: First-Mover Advantage and Industrial Location

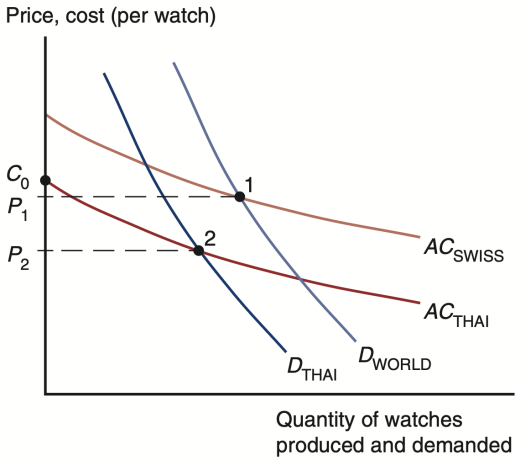


Figure 5: Potential Welfare Loss from Trade

6 Welfare Implications: Loss from Trade

Contrary to standard models, trade based on External EOS is not mutually beneficial in all cases. A country might be worse off than in autarky.

Example: Watches (Switzerland vs. Thailand)

- Switzerland is the established leader (First-mover).
- Thailand could be more efficient ($AC_{THAI} < AC_{SWISS}$).
- If trade is open, Switzerland supplies the world at P_1.
- If Thailand blocks trade (Autarky), its domestic industry could develop. Given sufficient domestic demand, the autarkic price might settle at P_2.
- If $P_2 < P_1$, Thailand pays *more* for watches under free trade than it would in autarky.

7 Dynamic Increasing Returns

Dynamic increasing returns refer to the reduction in costs over time as cumulative output increases (Learning-by-doing).

- **The Learning Curve:** Relates unit cost to cumulative output (L).

- **Infant Industry Argument:** Temporary protectionism may be justified to allow a domestic industry to accumulate the experience (Q_L) necessary to become competitive (C_1).

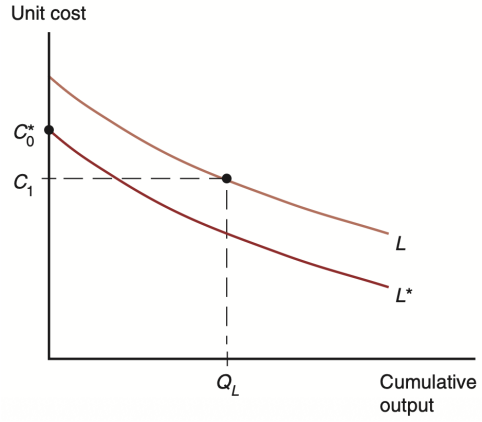


Figure 6: The Learning Curve

8 Inter-regional Trade and Economic Geography

- **Non-tradable Industries:** Must be supplied locally (e.g., haircuts, veterinary services). Employment is distributed roughly in proportion to population.
- **Tradable Industries:** Can concentrate disproportionately due to External EOS (e.g., finance in New York, tech in Silicon Valley). This shapes the economic geography of a country.

Instruments of Trade Policy

Ambrose W

1 Introduction to Tariffs

A tariff is the simplest trade policy instrument, defined as a tax levied when a good is imported. There are two primary types:

1. **Specific Tariff:** A fixed charge for each unit of imported goods (e.g., $3 per barrel of oil).
2. **Ad Valorem Tariff:** A fixed fraction of the value of the imported goods (e.g., 25% tariff on trucks).

2 Partial Equilibrium Analysis of Tariffs

To understand the effects of a tariff, we use a partial equilibrium framework. **Assumptions:**

- Perfect competition.
- Zero transportation costs (initially).
- Analysis focuses on a single market (ignoring indirect impacts on the wider economy).
- Prices in both markets are quoted in domestic currency.

2.1 Import Demand and Export Supply Curves

We derive two key curves to model trade between Home and Foreign.

2.1.1 Home Import Demand Curve (MD)

The Import Demand curve represents the excess demand in the Home market. It is defined as the difference between the quantity Home consumers demand and the quantity Home producers supply at each price.

$$MD = D - S \tag{1}$$

Characteristics:

- **Downward sloping:** As the price (P) increases, Home quantity demanded (D) falls and Home quantity supplied (S) rises, causing import demand (MD) to decline.
- Intersects the price axis at the Home autarky equilibrium price, P_A.

Figure 1: Derivation of the Home Import Demand Curve

2.1.2 Foreign Export Supply Curve (XS)

The Export Supply curve represents the excess supply in the Foreign market. It is defined as the difference between the quantity Foreign producers supply and the quantity Foreign consumers demand at each price.

$$XS = S^* - D^* \tag{2}$$

Characteristics:

- **Upward sloping:** As the price (P) increases, Foreign quantity supplied (S^*) rises and Foreign quantity demanded (D^*) falls, causing export supply (XS) to rise.
- Intersects the price axis at the Foreign autarky equilibrium price, P_A^*.

Figure 2: Derivation of the Foreign Export Supply Curve

2.2 World Equilibrium

In free trade, the world equilibrium occurs where Home import demand equals Foreign export supply.

$$MD = D - S = S^* - D^* = XS \tag{3}$$

At this intersection, we determine the world price P_W and the world trade quantity Q_W.

$$D + D^* = S + S^* \Rightarrow D^W = S^W \tag{4}$$

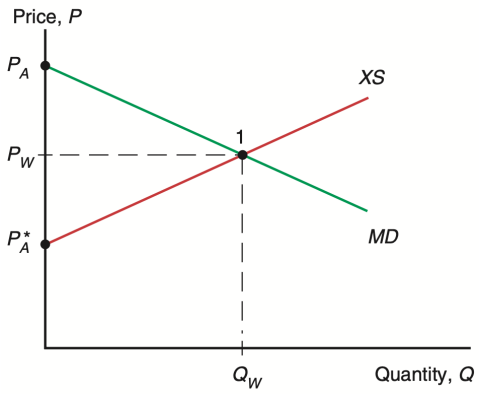


Figure 3: World Market Equilibrium

3 The Effect of a Tariff

A tariff acts like a transportation cost, creating a wedge between the price in the Home market and the Foreign market. Sellers will only ship goods if the price difference covers the tariff.

Pricing Conditions:

- **Specific Tariff (t):** $P_{Home} = P_{Foreign} + t$
- **Ad Valorem Tariff (τ):** $P_{Home} = (1 + \tau)P_{Foreign}$

3.1 Large Country Case

When a large country imposes a specific tariff t, it **affects world price**:

1. A wedge of size t opens in the world market: $P_T - P_T^* = t$.
2. **Home Price (P_T):** Rises, but by less than the full amount of the tariff ($P_T > P_W$).
3. **Foreign Price (P_T^*):** Falls ($P_T^* < P_W$). The tariff depresses the foreign export price.
4. **Trade Volume:** Declines from Q_W to Q_T.

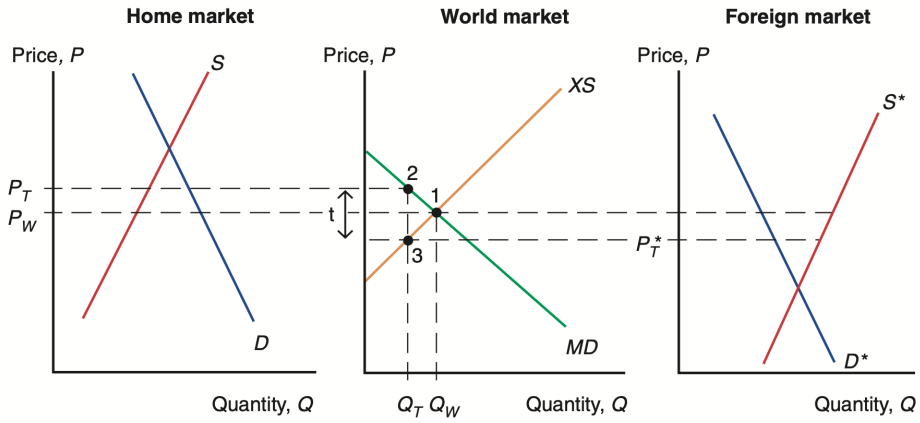


Figure 4: Effect of a Tariff on World Equilibrium

3.2 Small Open Economy (SOE) Case

A Small Open Economy has **no capability to affect world prices** because its demand is insignificant relative to the world.

- The Foreign (World) price remains unchanged at P_W.
- The Home price rises by the full amount of the tariff:

$$P_T = P_W + t \tag{5}$$

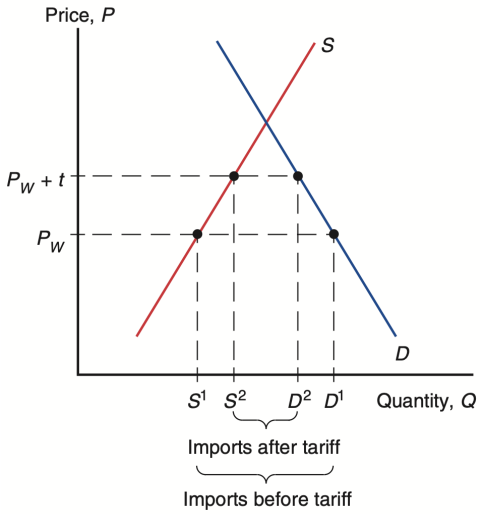


Figure 5: Effect of a Tariff in a Small Open Economy

4 Welfare Analysis of a Tariff

4.1 Definitions

- **Consumer Surplus:** Difference between the maximum willingness to pay and the price actually paid, summed over all units consumed.
- **Producer Surplus:** Difference between the price received and the minimum price at which producers are willing to sell, summed over all units produced.

4.2 Cost-Benefit Calculation (Home Country)

We analyze the welfare changes when the price rises from P_W to P_T (Home) and falls from P_W to P_T^* (Foreign).

1. **Domestic Producers (Winner):** Receive a higher price.

$$\Delta\text{Producer Surplus} = +a \tag{6}$$

2. **Domestic Consumers (Loser):** Pay a higher price and consume less.

$$\Delta\text{Consumer Surplus} = -(a + b + c + d) \tag{7}$$

3. **Government (Winner):** Collects tariff revenue.

$$\text{Gov Revenue} = t \times Q_T = (P_T - P_T^*) \times (D^2 - S^2) = c + e \tag{8}$$

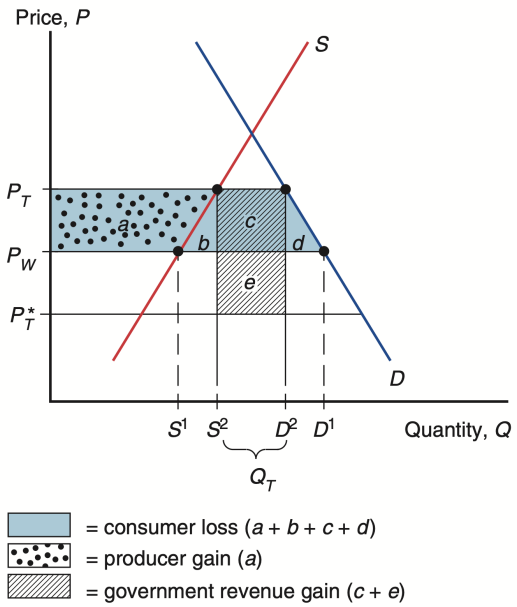


Figure 6: Welfare Costs and Benefits of a Tariff

4.2.1 Net Welfare Effect

The net cost is the sum of changes in consumer surplus, producer surplus, and government revenue.

$$\begin{aligned} \text{Net Cost of Tariff} &= \text{Consumer Loss} - \text{Producer Gain} - \text{Govt Gain} \\ &= (a+b+c+d) - a - (c+e) \\ &= b + d - e \end{aligned} \tag{9}$$

(Note: The standard formula represents the *cost*. A positive result indicates a loss, a negative result indicates a gain).

Interpretation of Areas:

- **Efficiency Loss ($b+d$):** The distortionary effect of the tariff.
  - Area b: Production distortion (Domestic production is too high).
  - Area d: Consumption distortion (Domestic consumption is too low).
- **Terms of Trade Gain (e):** The benefit from lowering foreign export prices ($P_T^* < P_W$).

$$ToT \uparrow = \frac{P_X}{P_M \downarrow} \tag{10}$$

Ambiguity: For a "large" economy, the effect is ambiguous. If $e > (b+d)$, the country gains. If $e < (b+d)$, the country loses. For a Small Open Economy, $e = 0$, so there is an unambiguous loss of $(b+d)$.

5 Other Instruments of Trade Policy

5.1 Export Subsidies

An export subsidy is a payment to a firm or individual who ships a good abroad. **Effect:**

- Home producers export until the domestic price exceeds the foreign price by the subsidy amount:

$$P_{Home} = P^* + \text{Subsidy} \tag{11}$$

- Raises Home price to P_S and lowers World price to P_S^*.

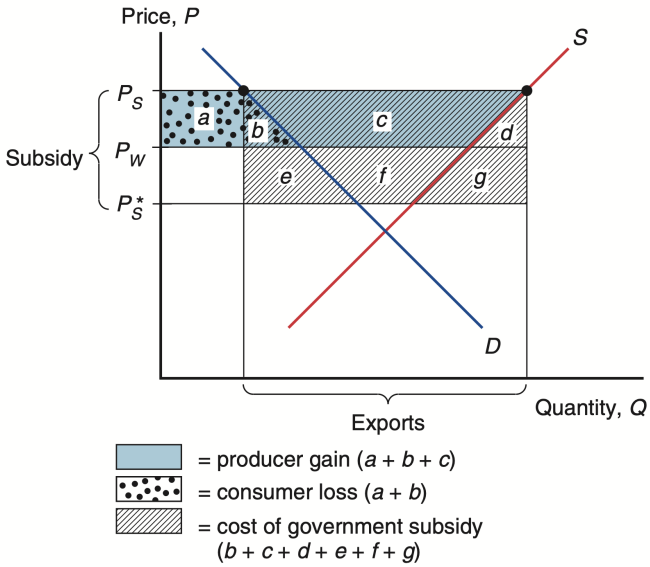


Figure 7: Welfare Effects of an Export Subsidy

Welfare Analysis:

- **Producer Gain:** $+(a+b+c)$
- **Consumer Loss:** $-(a+b)$
- **Cost of Subsidy:** $-(b+c+d+e+f+g)$
  - Note: Subsidy Cost = Exports $\times$ Subsidy $= (S^2 - D^2) \times (P_S - P_S^*)$.
- **Net Efficiency Loss:**

$$\text{Net Loss} = b + d + e + f + g \tag{12}$$

Conclusion: Export subsidies unambiguously lead to costs that exceed benefits ($b+d$ are distortion losses; $e+f+g$ are Terms of Trade worsening).

5.2 Import Quotas

A restriction on the quantity of a good that may be imported. **Mechanism (Small Country):**

- Restriction limits supply, raising price from P_W to P_Q.
- **Quota Rents (Area c):** The profit received by the holders of import licenses.

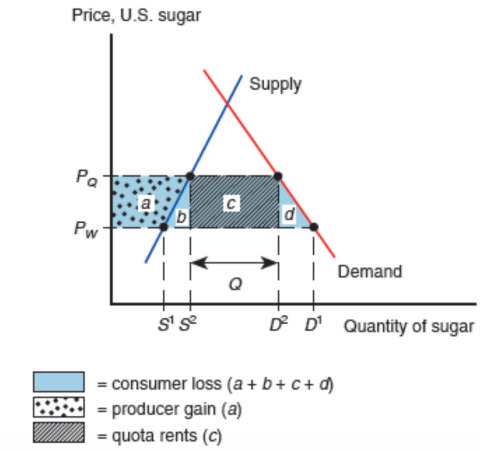


Figure 8: Welfare Effects of an Import Quota

Welfare Analysis:

- Consumer Loss: $-(a+b+c+d)$
- Producer Gain: $+a$
- Quota Rents: $+c$ (if retained by domestic license holders).
- **Net Societal Loss:** $b+d$ (if rents are domestic) or $b+c+d$ (if rents are lost to foreigners).

5.3 Voluntary Export Restraints (VER)

A quota on trade imposed from the *exporting* country (often at the importer's request).

- Works like a quota, but the **quota rents** are captured by foreign exporters.
- Always more costly to the importing country than a tariff.

5.4 Local Content Requirements

Requires a specified fraction (value or physical units) of a final good to be produced domestically.

- Does not generate government revenue or quota rents.
- Effective price of inputs becomes an average of imported and domestic prices.

Spatial Economies: Cities

Ambrose W

1 Introduction to Spatial Economies

Spatial economics is defined as the study of the allocation of resources over space and the location of economic activity. Unlike standard trade models which often treat countries as points, spatial economics explicitly considers distance, transport costs, and density.

Key Policy Questions:

- How to expand workers' access to the Central Business District (CBD) where the most productive jobs are located?
- What is the trade-off between housing investment versus transport infrastructure investment?

2 A Simple Spatial Model

This section outlines a closed, monocentric city model to understand location choices and market clearing.

2.1 Model Setup

- **Geography:** A closed city with a fixed supply of workers normalized to $L = 1$.
- **Employment:** Single work location (CBD). Workers are paid an exogenous wage w.
- **Housing:** Many residential locations (indexed by i) with land supply H_i.
- **Transportation:** Workers choose a commute mode m (e.g., car, subway).
- **Commuting Cost:** Defined as time lost. Let τ_i^m be the commute time. The effective labor endowment becomes $(1 - \tau_i^m)$.

2.2 Worker's Optimization Problem

Workers must choose:

1. Where to live (location i).
2. How to commute (mode m).
3. Consumption bundle (c vs h).

2.2.1 Utility Function

The worker's utility depends on consumption of goods (c), housing (h), commuting disutility, and an idiosyncratic preference shock:

$$U = c^{1-\alpha} h^{\alpha} \cdot (1 - \tau_i^m) \cdot \eta_i^m$$

Where:

- α: Share of expenditure on housing (Cobb-Douglas parameter).
- η_i^m: Idiosyncratic preference shock for location i and mode m.

2.2.2 Budget Constraint

Workers spend their wage w on tradeable goods (price normalized to 1) and housing rent r_i:

$$c + r_i h = w$$

2.2.3 Solving for Optimal Consumption

Conditional on choosing location i and mode m, the worker solves:

$$\max_{c,h} \quad c^{1-\alpha} h^{\alpha} \quad \text{s.t.} \quad c + r_i h = w$$

The Lagrangian:

$$\mathcal{L} = c^{1-\alpha} h^{\alpha} + \lambda(w - c - r_i h)$$

First Order Conditions (FOCs):

$$\frac{\partial \mathcal{L}}{\partial c} = (1-\alpha) c^{-\alpha} h^{\alpha} - \lambda = 0 \implies \lambda = (1-\alpha) c^{-\alpha} h^{\alpha}$$

$$\frac{\partial \mathcal{L}}{\partial h} = \alpha c^{1-\alpha} h^{\alpha-1} - \lambda r_i = 0 \implies \lambda r_i = \alpha c^{1-\alpha} h^{\alpha-1}$$

Dividing the second FOC by the first to eliminate λ:

$$r_i = \frac{\alpha c^{1-\alpha} h^{\alpha-1}}{(1-\alpha) c^{-\alpha} h^{\alpha}} = \frac{\alpha}{1-\alpha} \frac{c}{h}$$

Rearranging for c:

$$c = \frac{1-\alpha}{\alpha} r_i h$$

Substituting back into the budget constraint:

$$\frac{1-\alpha}{\alpha} r_i h + r_i h = w \implies r_i h \left(\frac{1-\alpha}{\alpha} + 1 \right) = w$$

$$r_i h \left(\frac{1}{\alpha} \right) = w$$

Optimal Demands:

$$h^* = \frac{\alpha w}{r_i} \quad ; \quad c^* = (1-\alpha) w$$

Note: Workers spend a constant fraction α of income on housing.

2.2.4 Indirect Utility

Substituting c^* and h^* back into the utility function:

$$V_{im} = ((1-\alpha)w)^{1-\alpha}\left(\frac{\alpha w}{r_i}\right)^{\alpha}(1-\tau_i^m)\eta_i^m$$

Grouping constants into a term $A = (1-\alpha)^{1-\alpha}\alpha^{\alpha}$:

$$V_{im} = A \cdot w \cdot r_i^{-\alpha} \cdot (1-\tau_i^m) \cdot \eta_i^m$$

Since A is constant across all choices, it does not affect the optimization. The relevant indirect utility is:

$$V_{im} \propto w r_i^{-\alpha}(1-\tau_i^m)\eta_i^m$$

2.3 Choice of Home Location and Mode

Workers choose (i, m) to maximize V_{im}.

- **Distributional Assumption:** The preference shock η is drawn from a Fréchet distribution with CDF $F(\eta) = e^{-\eta^{-\epsilon}}$.
- ϵ measures the homogeneity of preferences (inverse of variance). A higher ϵ means less variance in personal taste, making workers more sensitive to rent and commute times.

Choice Probability (ϕ_i^m): The probability of choosing location i and mode m is:

$$\phi_i^m = \frac{\left(w r_i^{-\alpha}(1-\tau_i^m)\right)^{\epsilon}}{\sum_k \sum_n \left(w r_k^{-\alpha}(1-\tau_k^n)\right)^{\epsilon}}$$

Let the denominator be the "Total City Attractiveness" Φ. Canceling w^{ϵ}:

$$\phi_i^m = \frac{r_i^{-\alpha\epsilon}(1-\tau_i^m)^{\epsilon}}{\Phi}$$

This implies:

- Higher Rent (r_i) $\rightarrow$ Lower Probability.
- Longer Commute (lower $1-\tau_i^m$) $\rightarrow$ Lower Probability.

2.4 Housing Market Equilibrium

Total spending on housing in neighborhood i must equal the value of housing stock supply.

$$r_i H_i = \alpha w \sum_m \phi_i^m$$

(Since total population $L = 1$).

3 Transport Networks and Welfare

3.1 Citywide Welfare

We group locations into sets with the same commute time. Welfare can be expressed as:

$$W = \sum_m \sum_{\kappa} (1-\tau(\kappa))^{\epsilon} \cdot \tilde{\Psi}^m(\kappa) \cdot \text{Area}^m(\kappa)$$

Where:

- $\text{Area}^m(\kappa)$: Total land area reachable within commute time κ using mode m.
- $\tilde{\Psi}^m(\kappa)$: Percentage of residents using mode m.

3.2 Accessibility Zones: US vs Europe

The lecture contrasts accessibility via Car vs. Public Transit in US and European cities.

Key Observations:

1. **Area:** For any travel duration, cars in the US cover a larger area than in Europe. However, public transit in Europe covers a larger area than in the US.
2. **General Rule:** Cars almost always provide greater accessibility (area coverage) than transit, except in very specific contexts like London.
3. **Population Density:** European cities are significantly more dense in the city center.
4. **Access to Population:** While cars reach more *land*, European public transit often provides better access to *population* due to high density.

Table: Average Accessibility Zone Areas by Region, Mode (km²)

	Car			Public Transit		
Min.	US	Europe	Ratio	US	Europe	Ratio
0-15	85.94	21.72	3.96	3.86	6.65	0.58
15-30	725.95	256.05	2.84**	29.70	61.17	0.49***
30-45	1493.27	863.23	1.73***	91.18	160.05	0.57***
45-60	2260.38	1702.59	1.33***	149.93	262.01	0.57***

Figure 1: Average Accessibility Zone Areas and Population Densities by Region and Mode

Policy Insights:

- Cars provide access to larger areas but generate negative externalities (congestion, health costs).
- Road investments require complementary land-use policies.

4 Quantitative Urban Models

To conduct realistic counterfactuals (e.g., building a subway), models must capture two opposing forces:

- **Agglomeration (Centripetal):** Increasing returns to scale, productivity spillovers.
- **Dispersion (Centrifugal):** Land scarcity (rents) and commuting costs.

4.1 The Economics of Density: Evidence from the Berlin Wall (Ahlfeldt et al., 2015)

Motivation: Identifying agglomeration is difficult due to omitted variable bias (natural advantages). The Berlin Wall serves as an exogenous source of variation in surrounding economic activity.

Model Specification:

- **Productivity (A_j):** Depends on fundamentals (a_j) and employment density (Υ_j):

$$A_j = a_j \Upsilon_j^{\lambda}$$

  Where λ is the agglomeration elasticity.

- **Amenities:** Depend on residential density (spillover effects).

Findings:

- **West Berlin:** Areas close to the Wall lost access to the historic CBD (East Berlin). This led to a sharp decrease in land prices and density in those areas.
- **Spillover Decay:** Productivity and amenity spillovers decline rapidly with travel time (near zero effect after approx. 20 minutes).
- **Conclusion:** A model without spillovers cannot explain the drastic rent changes observed. Agglomeration forces are required to match the data.

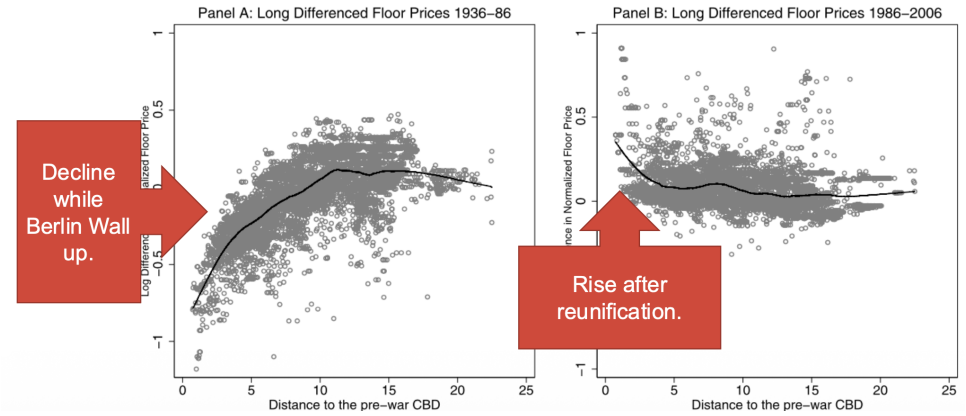


Figure 2: West Berlin Land Price Changes vs Distance to Pre-War CBD

4.2 The Making of the Modern Metropolis: London (Heblich et al., 2020)

Context: 19th Century London and the invention of the steam railway. **Question:** Did the separation of workplace and residence enabled by railways drive the concentration of economic activity?

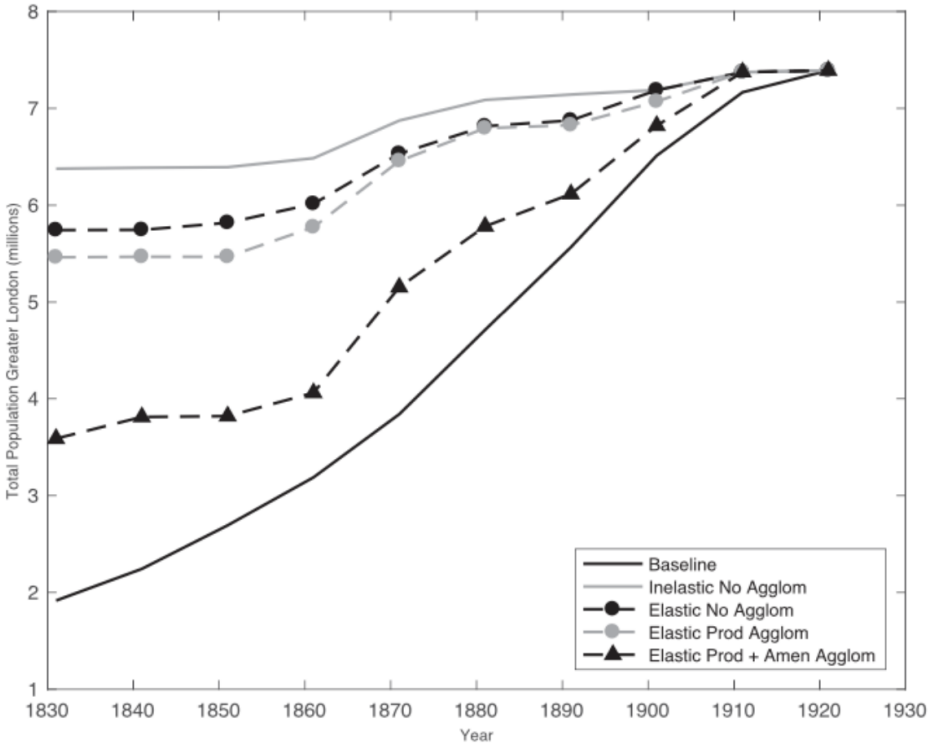
Methodology:

- **Backcasting:** Calibrated the model using 1921 bilateral commuting data and predicted backwards to 1831-1911.
- **Day vs. Night Population:** Historical data shows the City of London saw "Day Population" (employment) skyrocket while "Night Population" (residents) plummeted.

Key Results:

1. **Railway Effect:** Arrival of a station led to population increases in Outer London parishes and decreases in Central London (conversion to commercial space).
2. **Agglomeration Model:** To match historical data, the model *must* include both productivity spillovers (firms are more productive when clustered) and amenity spillovers.

3. **Mechanism:** Railways lowered commuting costs → Workers moved further out → Cluster drew labor from a larger catchment area → Increased cluster productivity → Higher wages → Further growth.



(C) Population Greater London (All Rail)

Figure 3: Model Prediction vs Historical Data: The Role of Agglomeration

5 References

- Ahlfeldt, G. M., Redding, S. J., Sturm, D. M., & Wolf, N. (2015). The economics of density: Evidence from the Berlin Wall. *Econometrica*, 83(6), 2127-2189.
- Heblich, S., Redding, S. J., & Sturm, D. M. (2020). The making of the modern metropolis: Evidence from London. *The Quarterly Journal of Economics*, 135(4), 2059-2133.

Spatial Economies II – Migration and Inequality

Ambrose W

1 The Basic Rosen-Roback Model

The Rosen-Roback model (Rosen 1979; Roback 1982) explains how wages and housing prices adjust to equalize utility across cities that differ in productivity and quality of life.

1.1 Core Assumptions

1. **Perfect Mobility:** Capital and labor are perfectly mobile across cities.
2. **Commuting:** Prohibitively high inter-city commuting costs (workers live where they work) but no intra-city commuting costs.
3. **Housing:** Fixed housing supply within cities; housing is interchangeable for residential or commercial use.
4. **Amenities (α):** Attributes of a place (climate, clean air) that are exogenous.
5. **Consumers are equally well-off across cities** as wage and housing cost compensate for intercity differences in amenity.
6. **Firms earn the same profit across cities** because renting cost and wage rate varies across cities

1.2 Worker's Problem

Consumers derive utility from consumption (c), housing (q), and urban amenities (α). The utility function is given by:

$$u(c, q, \alpha)$$

The price of consumption is normalized to 1 ($P_c = 1$). The worker's **Indirect Utility Function** represents the maximized utility achievable given income (y), housing price (p), and amenities (α):

$$V(y, p, \alpha)$$

- V is increasing in income (y) and amenities (α).
- V is decreasing in housing prices (p).

1.2.1 Worker Equilibrium Condition

In a spatial equilibrium, workers must be **indifferent between locations**. Therefore, utility must be equalized across all cities to a common level $\overline{u}$:

$$V(y, p, \alpha) = \overline{u}$$

The Indifference Curve: Plotting this in (y, p) space:

- The indifference curve is **upward-sloping**.

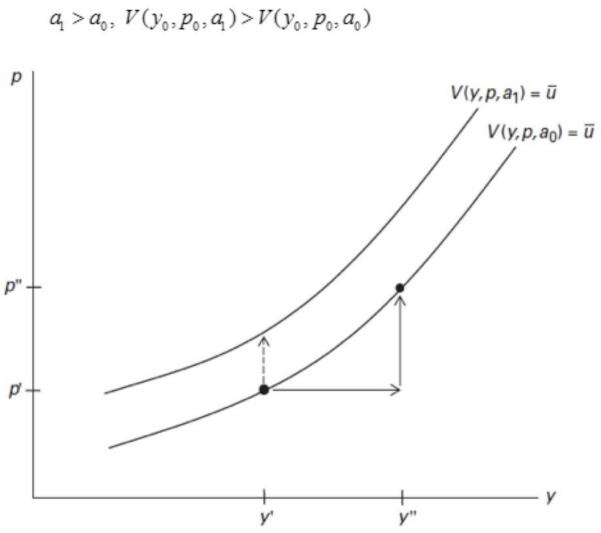


Figure 1: Indifference curves in (y, p) space.

- *Reasoning:* If income (y) rises, utility increases. To keep utility constant at $\overline{u}$, housing prices (p) must rise to offset the income gain.
- *Effect of Amenities:* For a higher amenity level $\alpha_1 > \alpha_0$, the indifference curve shifts. A location with better amenities provides higher utility, so for any given income y, the housing price p must be higher to maintain the equilibrium utility $\overline{u}$.

1.3 Firm's Problem

Firms produce non-housing goods (c) using labor and floor space. We assume Constant Returns to Scale (CRS).

- Cost of Labor = y (wages).
- Cost of floor space = p (rents).
- Amenities may affect costs (e.g., low crime reduces security costs; congestion increases delivery costs).

Unit Cost Function:

$$\text{Unit Cost} = C(y, p, \alpha)$$

1.3.1 Firm Equilibrium Condition

Because markets are perfectly competitive and firms are mobile, they must earn **zero profit in all regions**. Since the price of the output good is normalized to 1 ($c = 1$), the unit cost must equal the price:

$$C(y, p, \alpha) = 1$$

The Iso-profit Curve: Plotting this in (y, p) space:

- The iso-profit curve is **downward-sloping**.
- *Reasoning:* If labor costs (y) decrease, firms become profitable. To restore zero profit, housing/land costs (p) must increase.
- Effect of amenities:

– *Cost-reducing amenity:* The curve shifts upward (firms can afford higher rents/wages).
– *Cost-enlarging amenity:* The curve shifts downward.

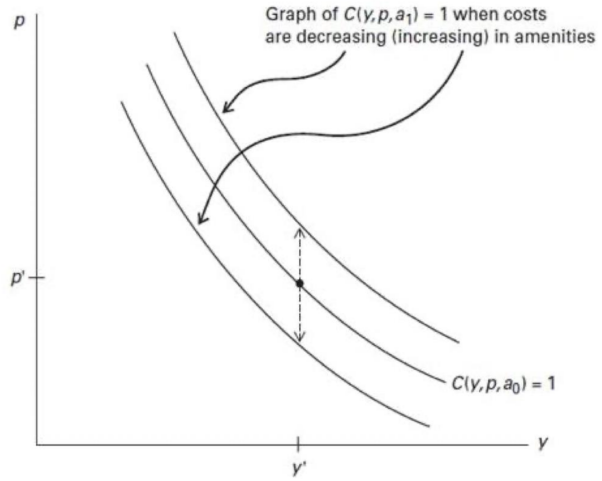


Figure 2: Firm Iso-profit curves in (y, p) space.

2 Spatial Equilibrium Analysis

The equilibrium for a specific city is determined by the intersection of the worker's indifference curve and the firm's iso-profit curve.

2.1 High vs. Low Amenity Regions

Consider two cities: City 0 (Low Amenity α_0) and City 1 (High Amenity α_1). Assume amenities affect consumer utility but do not affect firm costs (Common iso-profit curve).

- **Equilibrium 0:** Intersection of $IC(\alpha_0)$ and Common Iso-profit $\rightarrow (p_0, y_0)$.
- **Equilibrium 1:** Intersection of $IC(\alpha_1)$ and Common Iso-profit $\rightarrow (p_1, y_1)$.
- **Result:** Since $IC(\alpha_1)$ lies above $IC(\alpha_0)$ (workers accept lower wages/pay higher rents for better amenities), the high-amenity region is characterized by:
  - Higher housing prices ($p_1 > p_0$).
  - Lower nominal income ($y_1 < y_0$).

This lower wage is a *compensating differential* for the quality of life. Generally, Rosen-Roback predicts that **high-amenity region has higher housing price and lower income than low-amenity region**

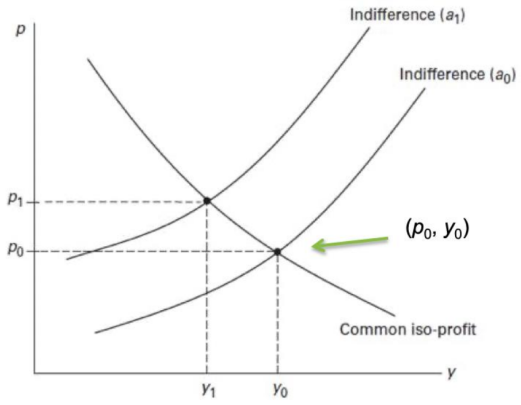


Figure 3: Equilibrium differences between high and low amenity regions.

3 Rosen-Roback w/ Worker Preference Shocks

We relax the assumption that workers are perfectly identical. Instead, workers have **idiosyncratic preferences** (preference shocks) for specific cities.

3.1 Setup

- **Real Wage (w):** Defined as nominal income minus housing costs:

$$w = y - p$$

- **Preference Shock:** The magnitude of a worker's preference for City A over City B. Higher "preference shock" means stronger preference for City A.
- **The Marginal Worker:** The worker who is indifferent between City A and City B.
  - Workers with preference shock > Marginal Worker: Live in City A.
  - Workers with preference shock < Marginal Worker: Live in City B.
  - All *inframarginal* workers are NOT indifferent between cities.

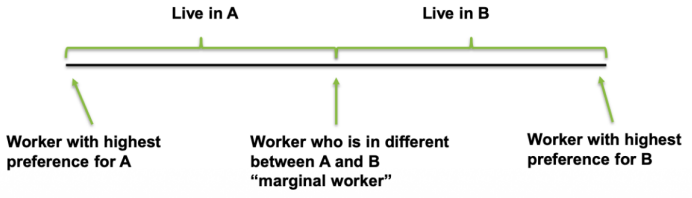


Figure 4: Sorting of workers based on preference shocks.

3.2 Comparative Statics: Productivity Shock

Suppose City A experiences a positive productivity shock (Nominal wage y increases, so w increases).

3.2.1 Mechanism

1. Initial state: Higher wage in A attracts inframarginal workers from B.
2. Migration: Population moves to A.
3. Housing Market: Demand for housing in A increases.
4. The outcome depends on the **Elasticity of Housing Supply**.

3.2.2 Scenario 1: Perfectly Inelastic Housing Supply

- Supply curve is vertical.
- Increased demand leads purely to higher prices ($p \uparrow$).
- **Result:** The rise in housing costs fully offsets the wage gain. Real wages do not change. Population does not grow in the long run.
- **Winner:** All benefits go to landowners in City A.

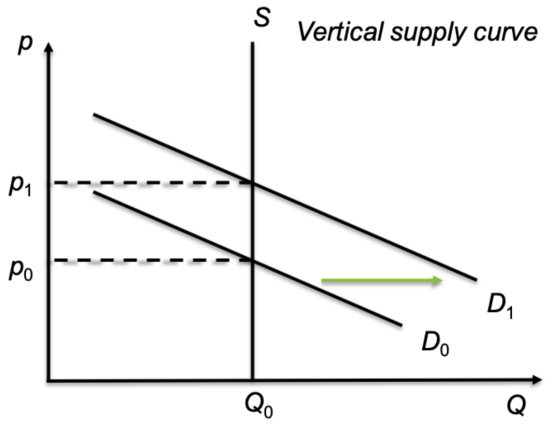


Figure 5: Effects of productivity shock under Inelastic housing supply

3.2.3 Scenario 2: Elastic Housing Supply

- Supply curve is upward sloping (or perfectly elastic).
- As population **moves to City A**, increased demand leads to higher quantity supplied ($Q \uparrow$) and a smaller increase in price ($p \uparrow$).
- **Effect on City A:** Housing prices rise less than the nominal wage increase. Real wages rise. Population in City A increases.
- **Effect on City B:** Out-migration reduces housing demand in B, lowering rents. Therefore, real wages in B also increase.
- **Results**: Workers in *both* cities benefit. Landowners in A benefit; Landowners in B lose

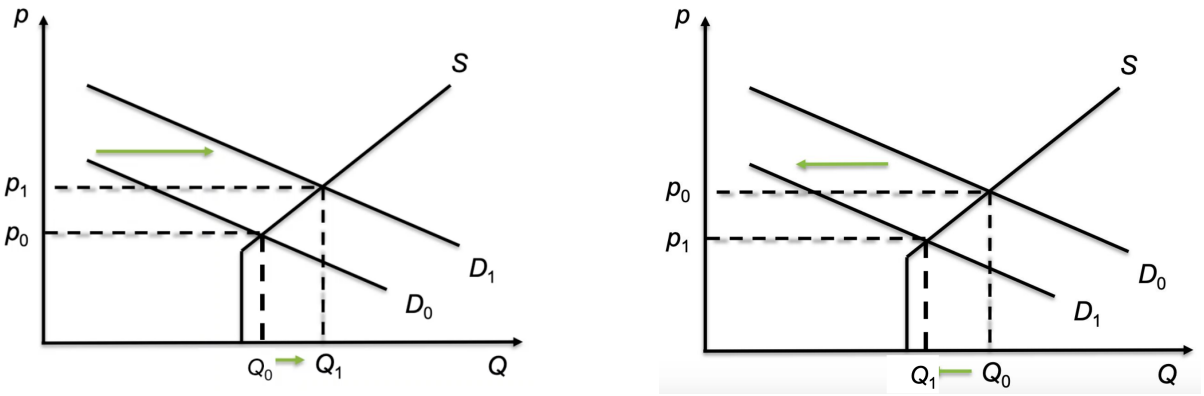


Figure 6: Effects of productivity shock under Elastic housing supply (Left: A; Right: B)

4 Empirical Evidence

4.1 The "Wrong Sign" Puzzle

The basic Rosen-Roback model predicts that high amenity areas should have high rents and *low* wages. However, data from Overman & Xu (2022) shows a positive correlation:

- **Observation:** Areas with high rents (e.g., London) also have high nominal wages.
- **Slope:** Positive slope in the plot of Log Monthly Rent vs. Area Wage Effect.

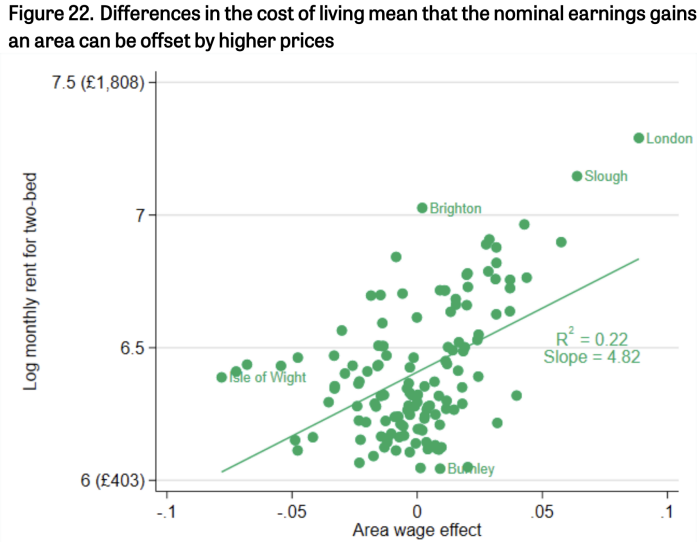


Figure 7: Log monthly rent vs Area wage effect (Overman & Xu, 2022).

4.2 Explaining the Deviation

The positive correlation suggests omitted variable bias. Two likely factors:

1. **Productivity Agglomeration:** Highly productive cities (London) generate sufficiently high wages that they offset both the high housing costs and any potential negative amenities (or complement positive ones).
2. **Amenity Production Effects:** If high amenities drastically *reduce* firm costs, firms can afford to pay higher wages.

4.3 Inferring Amenities

Since we cannot observe amenities directly, we infer them from the "Rent to Wage" relationship.

- **Logic:** If a place has high rent *relative* to wages, people must be accepting that lower real wage because the amenities are high.
- **Top Amenities:** Brighton, Bath, London.
- **Bottom Amenities:** Barnsley, Blyth.

4.4 Amenities and Human Capital

There is a strong positive correlation between implied amenities and the share of university graduates.

- **Conclusion:** Amenities are a key element contributing to whether a city can attract high-human-capital workers and develop high-tech clusters.