ECON0016 Term 2 - TNT Model

Ambrose W

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1 Time Frame

- Short-run: Supply of goods are rigid in the SR as labor is not occupationally mobile and technology is fixed. Therefore, relative prices & RER are primarily determined by demand factors in the SR
- Long-run: Labor are occupationally mobile and new technologies have an effect on production possibilities. Therefore, in the long-run, relative prices & RER are determined primarily by supply factors.

2 The TNT Model (Endowment Version)

Note: Basically a modified Chapter 3 model.

2.1 Environment and Preferences

Two goods: tradable (T) and non-tradable (N). Households maximize

$$U = \log C_1 + \beta \log C_2, \tag{1}$$

with Cobb–Douglas composite consumption:

$$C_1 = (C_1^T)^\gamma (C_1^N)^{1-\gamma}, \tag{2}$$
$$C_2 = (C_2^T)^\gamma (C_2^N)^{1-\gamma}, \tag{3}$$

where $\gamma \in [0, 1]$ is the (constant) expenditure share on tradables. In equilibrium the tradables expenditure share equals γ.

2.2 Budget Constraints and Relative Prices

Exogenous endowments (Q_t^T, Q_t^N); bond B_1 is in tradable units. Using $p_t \equiv P_t^N / P_t^T$:

$$C_1^T + p_1 C_1^N + B_1 = Q_1^T + p_1 Q_1^N,$$
$$C_2^T + p_2 C_2^N = Q_2^T + p_2 Q_2^N + (1 + r_1) B_1.$$

Eliminating B_1 yields the *intertemporal budget constraint*:

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1 + r_1} = Q_1^T + p_1 Q_1^N + \frac{Q_2^T + p_2 Q_2^N}{1 + r_1} \equiv Y \tag{4}$$

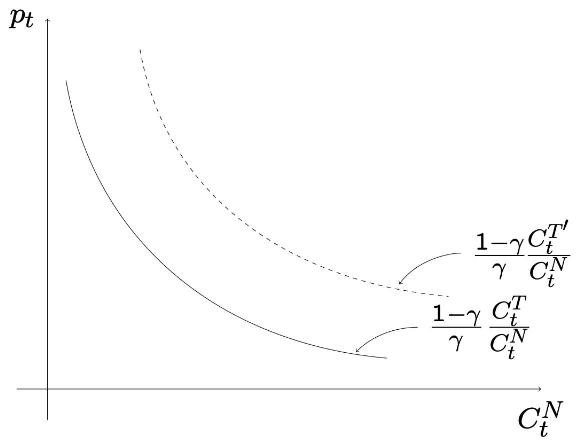
Note Y is no longer a constant as it depends on endogenous p_1, p_2 and r_1.

2.3 Household Problem and First-Order Conditions

Maximize (1) subject to (2)–(4). The FOCs imply:

$$C_2^T = \beta(1 + r_1)C_1^T, \qquad \text{(Euler equation)} \tag{5}$$

$$C_t^N = \frac{1-\gamma}{\gamma}\frac{C_t^T}{p_t}, \quad t = 1, 2. \qquad \text{(Demand for nontradables)} \tag{6}$$



2.4 Equilibrium with Free Capital Mobility

Nontradable market clears: $C_t^N = Q_t^N$, $t = 1, 2$. With free capital mobility:

$$r_1 = r^\star.$$

Using $C_t^N = Q_t^N$ in (4) delivers the *intertemporal resource constraint*:

$$C_1^T + \frac{C_2^T}{1+r^\star} = Q_1^T + \frac{Q_2^T}{1+r^\star}.$$

Combining with Euler Equation (5):

$$C_1^T = \frac{1}{1+\beta}\left(Q_1^T + \frac{Q_2^T}{1+r^\star}\right). \tag{8}$$

Hence $C_1^T = C^T(r^{\star-}, Q_1^{T+}, Q_2^{T+})$ as in (9). The trade balance and current account (with $B_0 = 0$) are

$$TB_1 = Q_1^T - C_1^T, \qquad CA_1 = TB_1.$$

2.5 Equilibrium Relative Price of Nontradables and Comparative Statics

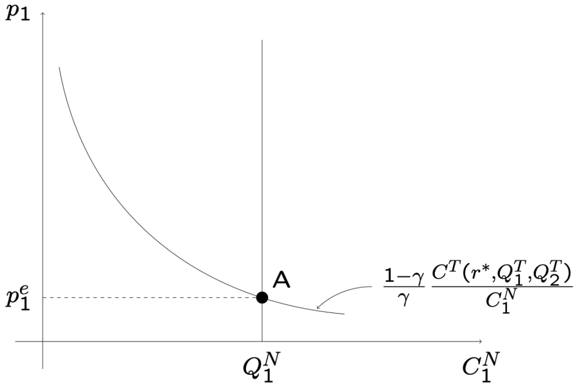
From (6), market clearing $C_1^N = Q_1^N$ gives the equilibrium demand function of non-tradables:

$$Q_1^N = \frac{1-\gamma}{\gamma}\frac{C_1^T}{p_1} = \frac{1-\gamma}{\gamma}\frac{C^T(r^{\star-}, Q_1^{T+}, Q_2^{T+})}{p_1}. \tag{10}$$

Solving pins down $p_1 = p(r^{\star-}, Q_1^{T+}, Q_2^{T+}, Q_1^{N-})$. Collecting signs:

$$p_1 = p\Big(r^{\star-},\, Q_1^{T+},\, Q_2^{T+},\, Q_1^{N-}\Big). \tag{11}$$

Recall that supply of non-tradables is perfectly inelastic, we have the SD Diagram. An equilibrium is achieved when the demand curve intersects the supply curve at the **Equilibrium value of the relative price of non-tradables** (p_1^e).



- Interest rate effect ($r^\star \uparrow$): Negative, shifts the demand curve down, $p_1^e \downarrow$ (Real depreciation)
- Transitory increase in the endowment of tradable: Positive, shifts the demand curve up, $p_1^e \uparrow$ (Real appreciation)
- Expected future increase in endowment of tradable: Positive, shifts the demand curve up, $p_1^e \uparrow$ (Real appreciation)
- Increase in the endowment of non-tradable: Negative, movement along the demand curve, $p_1^e \downarrow$ (Real depreciation)

3 From p_t to the Real Exchange Rate

Let the price aggregator be homothetic; under Cobb–Douglas aggregator (1) and (2):

$$P_t = \phi(P_t^T, P_t^N) = (P_t^T)^\gamma (P_t^N)^{1-\gamma} A, \quad A = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}.$$

$$P_t^\star = \phi(P_t^{T\star}, P_t^{N\star})$$

The real exchange rate is e_t, and by homogeneity plus the law of one price (LOOP) for tradables ($\epsilon_t P_t^{T\star} = P_t^T$):

$$e_t = \epsilon_t \frac{P_t^\star}{P_t} = \epsilon_t \frac{\phi(P_t^{T\star}, P_t^{N\star})}{\phi(P_t^T, P_t^N)} = \frac{\epsilon_t P_t^{T\star}\, \phi\left(1, \frac{P_t^{N\star}}{P_t^{T\star}}\right)}{P_t^T\, \phi\left(1, \frac{P_t^N}{P_t^T}\right)} = \frac{\phi^\star(1, p_t^\star)}{\phi(1, p_t)}.$$

Thus, given $p_t^\star$, the RER is a decreasing function of p_t. Combining with (11) yields

$$e_1 = e\left(r^{\star +}, Q_1^{T-}, Q_2^{T-}, Q_1^{N+}, p_1^{\star +}\right). \tag{12}$$

4 Terms of Trade (ToT) and the RER

Suppose all consumptions of tradables are imported and all endowment of tradables are exported. Recall in Chapter 3 that we must replace Q_t^T by $TT_t\, Q_t^T$, then

$$C_1^T = C^T\big(\underset{-}{r^*}, \underset{+}{TT_1 Q_1^T}, \underset{+}{TT_2 Q_2^T}\big),$$

$$C_1^N = \frac{1-\gamma}{\gamma} \cdot \frac{C^T\big(\underset{-}{r^*},\, \underset{+}{TT_1 Q_1^T},\, \underset{+}{TT_2 Q_2^T}\big)}{\underset{-}{p_1}}.$$

$$p_1 = p\Big(r^{\star -},\, (TT_1 Q_1^T)^+,\, (TT_2 Q_2^T)^+,\, Q_1^{N-}\Big),$$

$$e_1 = e\big(\underset{+}{r^*},\, \underset{-}{TT_1 Q_1^T},\, \underset{-}{TT_2 Q_2^T},\, \underset{+}{Q_1^N},\, \underset{+}{p_1^*}\big).$$

and an improvement in TT_1 or TT_2 raises p_1 (nontradables become relatively expensive) and appreciates the RER (lowers e_1).

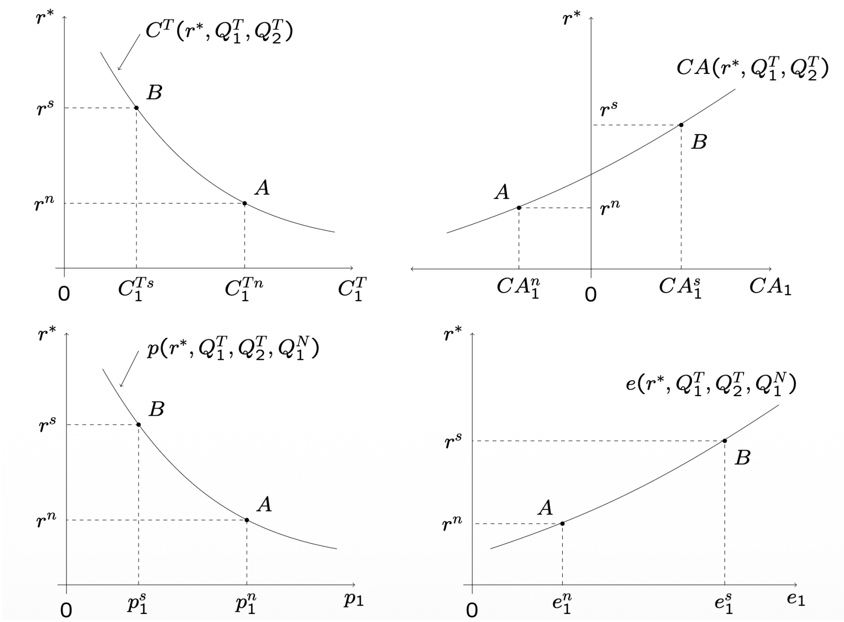
5 Sudden Stops

5.1 Definition and Stylized Facts

A sudden stop is a sharp loss of external credit access, modeled as an increase in $r^\star$. Typical outcomes: (i) current account reversal (or large deficit reduction), (ii) contraction in aggregate demand, (iii) real depreciation.

5.2 Mechanism in the TNT Model

- $C_1^T \downarrow$ as in (8), due to negative wealth effect from higher $r^\star$.
- $CA \uparrow$ as $TB_1 \uparrow = Q_1^T - C_1^T \downarrow$ :: Current account reversal from deficit to surplus.
- $p_1 \downarrow$ as in (11) because relative price of non-tradables must be cheaper to clear the non-tradables market with lower desired level of C_1^N.
- $e_1 \uparrow$ as in (12): Real depreciation
- Hence, we have saw:
  - Expenditure switching from tradables to non-tradables.
  - Contraction in AD
  - Current account reversal
  - Real depreciation (i.e. country becomes less expensive to foreigners)



6 TNT Model with Sectoral Production

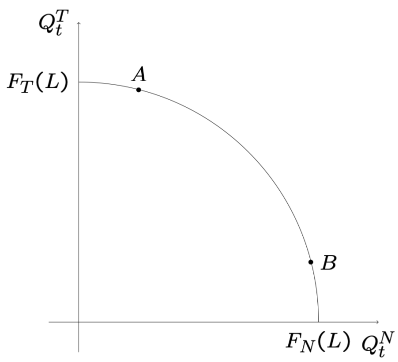
6.1 Technology, Feasible Set, and PPF

Sectoral outputs depend on labor inputs:

$$Q_t^T = F_T(L_t^T), \qquad Q_t^N = F_N(L_t^N), \qquad L_t^T + L_t^N = L,$$

with $F_i' > 0$, $F_i'' < 0$. Substitute production functions into labor resource constraint, we have the Production Possibility Frontier (PPF):

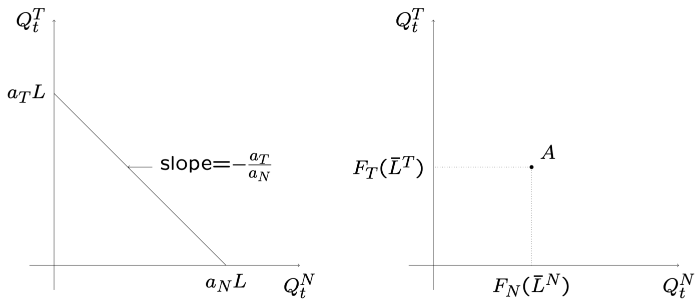
$$\underbrace{F_T^{-1}Q_t^T}_{L_t^T} + \underbrace{F_N^{-1}Q^N}_{L_t^N} = L$$



The PPF slope follows from marginal rates of transformation (MRT):

$$\frac{dQ_t^T}{dQ_t^N} = -\frac{F_T'(L_t^T)}{F_N'(L_t^N)}.$$

6.1.1 Special Cases



- (LHS) Linear Production Function: $Q_t^T = a_T L_t^T$ and $Q_t^N = a_N L_t^N$
- (RHS) Full Labor Specialization: Every worker is perfectly occupationally immobile (i.e. only work in one sector).

6.2 Firms' Static Optimality and the p_t–PPF Tangency

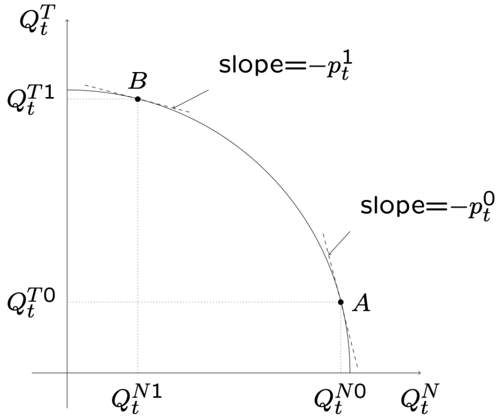
Firms choose L_t^T to maximize profits:

$$\Pi_t^T = P_t^T F_T(L_t^T) - W_t L_t^T,$$
$$\Pi_t^N = P_t^N F_N(L_t^N) - W_t L_t^N,$$

The first order conditions imply:

$$\frac{F_T'(L_t^T)}{F_N'(L_t^N)} = \frac{P_t^N}{P_t^T} \equiv p_t. \tag{1}$$

Thus, the profit-maximizing production point is the tangency of the PPF with the relative price line of slope $-p_t$.



6.3 Households, Budget, and the Same Intertemporal Structure

With labor income $w_t L$ and sectoral profits Π_t^T, Π_t^N, the period constraints are

$$P_1^T C_1^T + P_1^N C_1^N + P_1^T B_1 = w_1 L + \Pi_1^T + \Pi_1^N,$$
$$P_2^T C_2^T + P_2^N C_2^N = w_2 L + \Pi_2^T + \Pi_2^N + (1+r_1) P_2^T B_1.$$

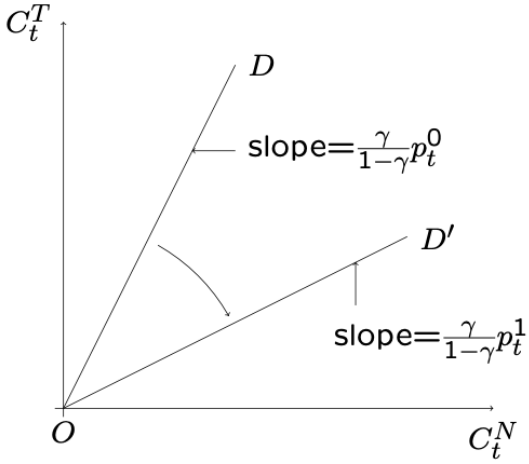
Divide Period 1 constraint by P_1^T and Period 2 constraint by $P_2^T(1+r_1)$, the IBC is

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1+r_1} = \underbrace{\frac{W_1 L + \Pi_1^T + \Pi_1^N}{P_1^T} + \frac{w_2 L + \Pi_2^T + \Pi_2^N}{P_2^T(1+r_1)}}_{\bar{Y}}$$

Given that households take $\bar{Y}$ as given, the FOCs are identical to the endowment version:

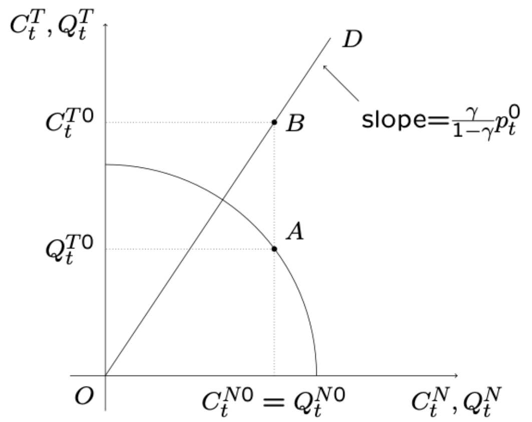
$$C_2^T = \beta(1+r_1) C_1^T,$$
$$C_t^T = \frac{\gamma}{1-\gamma} p_t C_t^N, \quad t = 1,2 \quad \text{[Income Expansion Path]}.$$

whose slope increases in p_t.



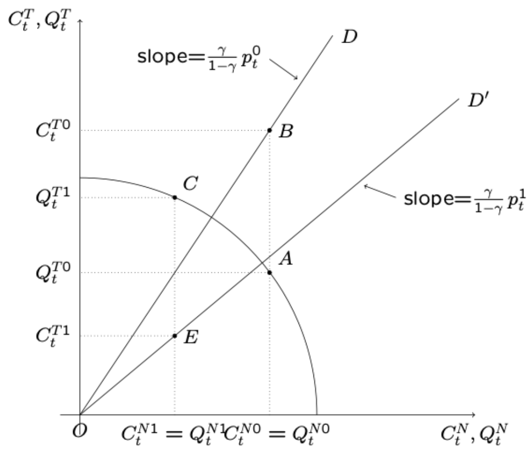
6.4 Partial-Equilibrium Mapping at a Given p_t

Partial Equilibrium: Taking an *endogenous* variable as given to analyze the behavior of other endogenous variables.
Treat p_t^0 as given (i.e. Slope of PPF $= -p_t^0$), we have partial equilibrium:



- Production: PPF tangency is Point A with (Q_t^{T0}, Q_t^{N0}).
- Market clearing: $C_t^{N0} = Q_t^{N0}$.
- IEP: Given C_t^{N0}, we have corresponding C_t^{T0} at Point B.
- Trade Balance: Difference between production and consumption of tradables is $TB_t = Q_t^{T0} - C_t^{T0}$. In this case, the country must run a trade deficit.

6.4.1 Effect of a RER depreciation ($p_t \downarrow$)



Before Depreciation:

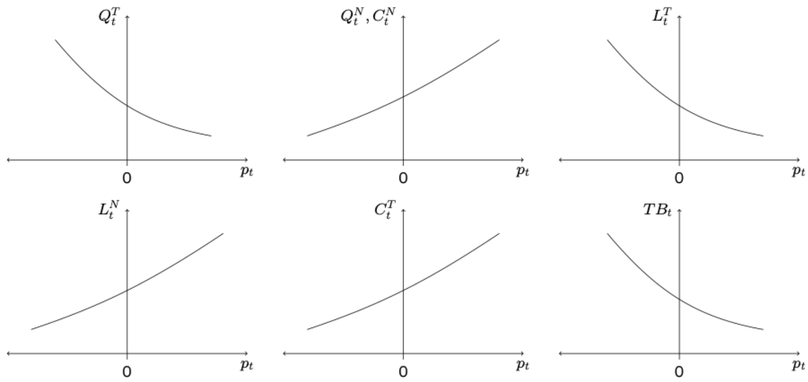
- Economy produces at Point A and consume Point B
- Trade balance of $TB_t = Q_t^{T0} - C_t^{T0} < 0$.

After RER Depreciation:

- Economy produces at Point C (Less non-tradable)& IEP shifts clockwise
- Market clearing: $C_t^{N1} = Q_t^{N1}$

- IEP: Given C_t^{N1}, we have corresponding C_t^{T1} at Point E.
- Trade balance: Now $Q_t^{T1} > C_t^{T1}$, hence we have a trade surplus!

6.4.2 Relationship under Partial Eq: Endogenous variable as functions of p_t



7 General Equilibrium with Sectoral Production

We now consider the IRC and Euler Equation to establish general equilibrium.

7.1 Two Loci in (TB_1, TB_2) Space

7.1.1 Euler Locus (EE)

Because C_t^T is a decreasing function of TB_t in partial equilibrium, write $C_t^T = C^T(TB_t)$. The **Euler Locus (EE)** is:

$$C^T(TB_2) = \beta(1+r^\star)C^T(TB_1)$$

7.1.2 Intertemporal Resource Constraint Locus (II)

We use the market clearing condition of non-tradables and definition of profits to rewrite with-in period constraints:

$$C_1^T + B_1 = Q_1^T$$
$$C_2^T = Q_2^T + (1+r^\star)B_1$$

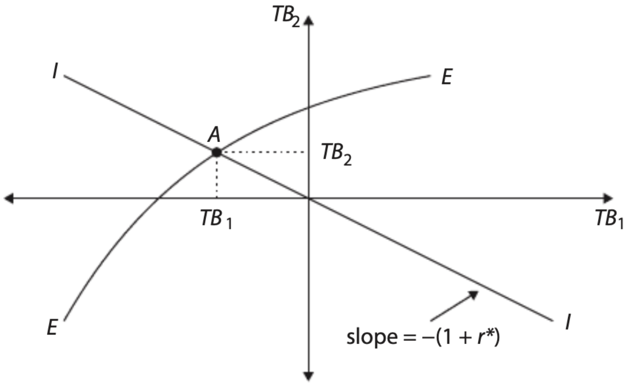
which combine to

$$C_1^T + \frac{C_2^T}{1+r^\star} = Q_1^T + \frac{Q_2^T}{1+r^\star} \quad \iff \quad TB_2 = -(1+r^\star)\,TB_1.$$

Hence, IRC Locus is downward sloping and pass through the origin (as we assume $B_0 = 0$). If $B_0 < 0$, then II have a positive intercept, vice versa.

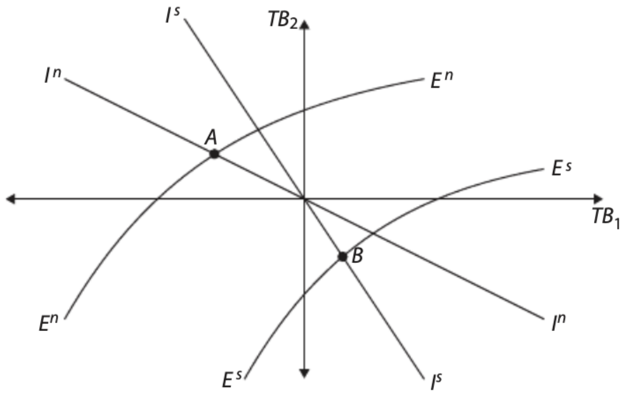
7.2 General Equilibrium Solution

The intersection of EE and II determines (TB_1, TB_2) in general equilibrium. We assume $\beta(1+r^\star) < 1$, which implies $C_2^T < C_1^T$ and $TB_2 > TB_1$,so that EE have a positive intercept.



7.3 Sudden Stop and Sectoral Reallocation

Suppose world interest rate $r^n \to r^s > r^n$



Trade balance effect:

- Euler Equation: $E^n \to E^s$
  - Assume $\beta(1 + r^S) > 1$ implies $C^T(TB_2) > C^T(TB_1) \implies TB_2 < TB_1$ as $C^T(TB_t)$ is strictly decreasing, then the intercept of locus E^s becomes negative
  - EE shifts downward and to the right
- IRC Locus: $I^n \to I^s$, a clockwise rotation
- Equilibrium: Point A $\to$ Point B, suggesting current account reversal from deficit to surplus

General equilibrium effect:

- Contraction in AD ($C_1^T \downarrow, C_2^T \downarrow$)
- RER Depreciation ($e_1 \downarrow$) as collapse in AD drives down $p_1 \downarrow$
- Reallocation of output and employment away from non-traded sector
- Involuntary unemployment.