ECON0016 Term 2 - Beyond saving version...

Ambrose W

August 10, 2025

1 Chapter 1: Global Imbalances

1.1 Overview

Global imbalances refer to the persistent current account (CA) deficits and surpluses observed across countries. Notably, since the 1980s, the United States has emerged as the world's largest net debtor, while countries like China, Japan, and Germany have become major net creditors. These imbalances necessitate borrowing by deficit countries from surplus countries.

1.2 Balance of Payments Accounts

The balance of payments records a country's international transactions and comprises the Current Account, the Financial Account, and the Capital Account.

1.2.1 Current Account (CA)

The CA records non-asset exchange transactions.

$$CA \text{ Balance} = \text{Trade Balance} + \text{Income Balance} + \text{Net Unilateral Transfers}$$

- **Trade Balance (Net Exports):** Difference between exports and imports of goods and services.
- **Income Balance (Primary Income):** Difference between income received from abroad (e.g., investment income, compensation of employees) and income paid to foreigners.
- **Net Unilateral Transfers (Secondary Income):** Payments without a corresponding exchange of goods/services (e.g., foreign aid, remittances).

For the U.S., the CA balance generally follows the trade balance, with primary income being a significant positive component (net investment income) and secondary income typically negative (net outflows like remittances).

1.2.2 Financial Account (FA)

The FA records transactions involving the exchange of assets.

$$FA \text{ Balance} = \text{Increase in foreign-owned assets in country} - \text{Increase in country-owned assets abroad}$$

It includes Foreign Direct Investment (FDI) (lasting interest, $\geq 10\%$ ownership) and Portfolio Investment (financial assets like stocks/bonds, $\leq 10\%$ ownership).

1.2.3 Capital Account

This account records minor transactions like debt forgiveness or transfer of assets by migrants. It is often grouped with the FA or considered negligible for most economies in this context.

1.3 Net International Investment Position (NIIP)

The NIIP represents the difference between the value of foreign assets owned by domestic residents and the value of domestic assets owned by foreigners. It is a stock variable reflecting a country's net foreign wealth or indebtedness.

- $NIIP > 0$: Net creditor
- $NIIP < 0$: Net debtor

The change in NIIP is influenced by the CA balance and valuation changes in assets and liabilities.

$$\Delta NIIP_t = CA_t + \text{Valuation Changes}_t$$

Valuation changes arise from fluctuations in exchange rates and asset prices. These effects have become more pronounced due to the explosion in gross international asset and liability positions since the 1990s. For the U.S., significant valuation gains, particularly during periods of dollar depreciation or strong foreign market performance, have mitigated the decline in its NIIP despite persistent CA deficits.

1.4 The NIIP 'Paradox'

The phenomenon where a debtor country like the U.S. consistently earns positive net investment income is sometimes termed a paradox. Explanations include:

1. **Underestimation of U.S. Foreign Assets:** Official statistics may not capture intangible assets (e.g., brand capital, entrepreneurial know-how) held abroad, particularly through FDI, which generate income flows recorded in the CA. This suggests the "true" NIIP might be less negative than reported.

2. **Rate of Return Differential:** The U.S. tends to earn higher returns on its foreign assets (often riskier assets like FDI and equity) than foreigners earn on their U.S. assets (often safer assets like Treasury bonds). A simple model illustrates this:

$$NIIP_t = r^{FA} A_t - r^{FL} L_t \quad \text{(Net Investment Income)}$$

   If $r^{FA} > r^{FL}$, positive net income can coexist with $NIIP < 0$ (i.e., $A_t < L_t$).

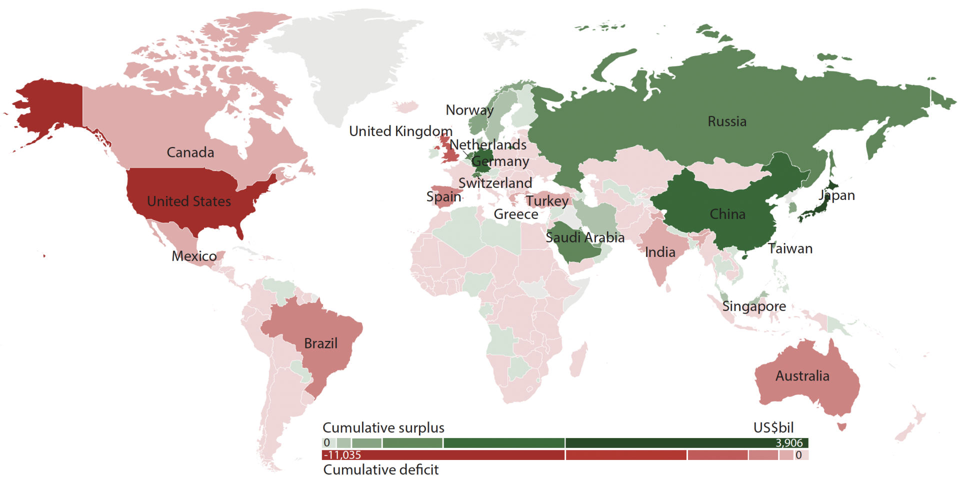


Figure 1.1. Cumulative Current Account Balances around the World: 1980–2017

Figure 1: Global Current Account Imbalances.

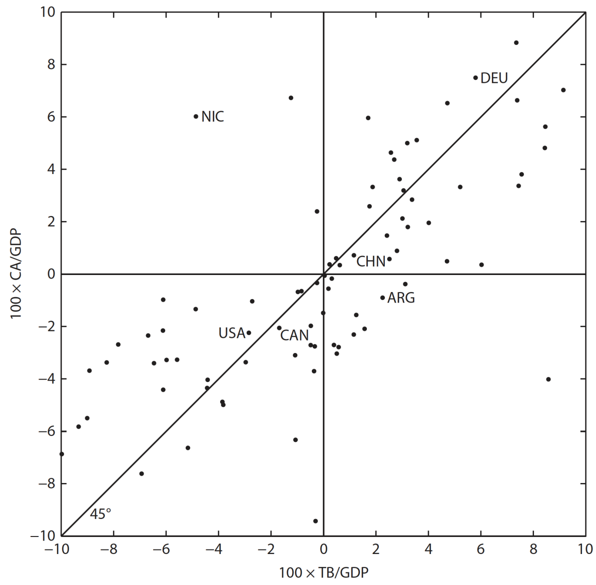


Figure 2: U.S. Trade Balance and Current Account (% of GDP).

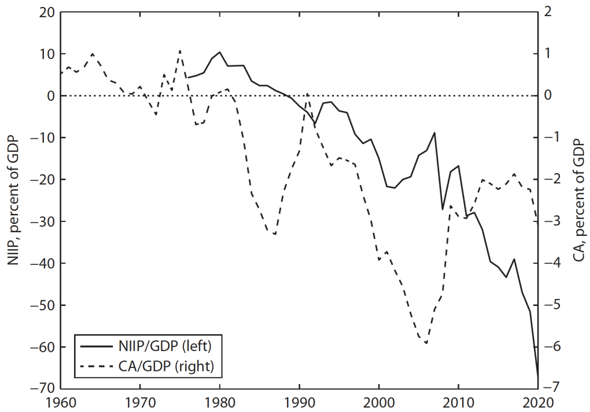


Figure 3: Evolution of U.S. NIIP.

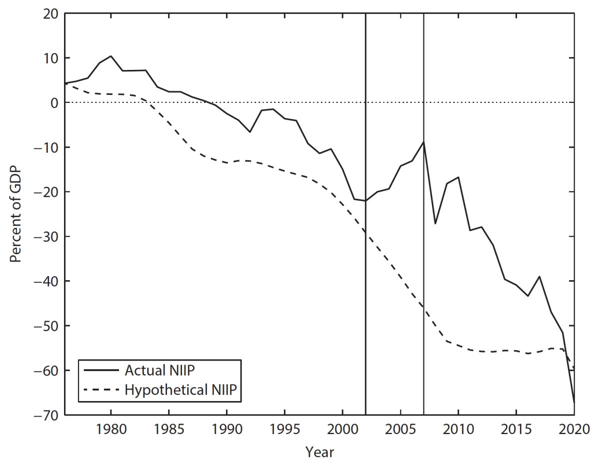


Figure 4: Actual vs. Hypothetical U.S. NIIP (Excluding Valuation Changes).

2 Chapter 2: Current Account Sustainability

2.1 The Intertemporal Budget Constraint (IBC)

A country's spending must be consistent with its ability to repay debts over time.

2.1.1 Basic Framework (Two Periods)

Let $NIIP_t$ be the net international investment position at the end of period t. B_t denotes net foreign assets held by the country at the end of period t (so $NIIP_t = B_t$). Let r_t be the world interest rate on assets held from $t-1$ to t. TB_t is the trade balance and CA_t is the current account balance in period t.

The change in net foreign assets is driven by the current account:

$$B_t - B_{t-1} = CA_t \quad \text{(Rearranged from Eq. 2)}$$

The current account comprises the trade balance and net investment income:

$$CA_t = TB_t + r_{t-1}B_{t-1} \quad \text{(Simplified from Eq. 1)}$$

Combining these gives the evolution of net foreign assets:

$$B_t = (1 + r_t)B_{t-1} + TB_t$$

For a two-period model (t=1, 2), starting with B_0 (initial NIIP), we have:

$$B_1 = (1 + r_1)B_0 + TB_1$$
$$B_2 = (1 + r_2)B_1 + TB_2$$

2.1.2 Transversality Condition (No Ponzi Games)

No-Ponzi-Game condition necessitates that the country cannot run a trade deficit indefinitely without repaying its debts. This implies that at the end of the last period, net foreign assets must be zero $B_2 = 0$:

2.2 Derivation & Trade deficit sustainability

Substituting B_1 into the equation for B_2 and setting $B_2 = 0$:

$$0 = (1 + r_2)((1 + r_1)B_0 + TB_1) + TB_2$$

Rearranging yields the Intertemporal Budget Constraint (IBC):

$$(1 + r_1)(1 + r_2)B_0 + (1 + r_2)TB_1 + TB_2 = 0$$

Or, in present value terms (dividing by $(1+r_1)(1+r_2)$ and assuming $r_1 = r_2 = r^*$ for simplicity):

$$(1 + r^\star)B_0 = -TB_1 - \frac{TB_2}{1 + r^\star}$$

Or more generally:

$$B_0 = -PV(\text{Future Trade Balances}) = -\sum_{t=1}^{\infty} \frac{TB_t}{(1 + r^*)^t}$$

Implication:

- $B_0 < 0 \implies$ CANNOT run perpetual trade deficit
- $B_0 > 0 \implies$ CAN run perpetual trade deficit

2.3 Current Account Deficit Sustainability

Can a country run a CA deficit forever?

- $B_0 > 0$ allows the country to run a perpetual CA deficit.
- $B_0 < 0$ implies the country CANNOT run a CA deficit in all finite future periods. However, it does not *necessarily* preclude it from running perpetual CA deficit in an *infinite* horizon.

2.4 Gross National Product

Gross National Product (GNP) is the total income earned by a country's residents, including income from abroad, minus income earned by foreigners within the country. It can be expressed as:

$$Y_t = GDP + NII = Q_t + rB_{t-1} = C_t + G_t + I_t + TB_t + rB_{t-1}$$

2.5 Domestic Absorption

$$A_t = C_t + I_t + G_t$$

2.6 Current Account, Saving, and Investment

- National Saving = National Income (GNP) - Private & Public consumption

Current account identity:

$$CA_t = S_t - I_t = (Y_t - C_t - G_t) - I_t = Y_t - A_t$$

Implication: A CA deficit ($CA_t < 0$) implies that domestic investment exceeds national saving ($I_t > S_t$), financed by capital inflows (borrowing from abroad). A CA surplus ($CA_t > 0$) means national saving exceeds domestic investment ($S_t > I_t$), leading to capital outflows (lending abroad).

3 Chapter 3: An Intertemporal Theory of the Current Account

3.1 Model Setup: Small Open Endowment Economy

We analyze a small open economy (SOE), which is an endowment economy with no production or investment decisions. The economy is characterized by:

- **(Sufficiently) Small Open Economy (SOE):** Takes the world interest rate (r^*) as given; trade cannot influence world prices.
- **Endowment Economy:** Representative & utility-maximizing households receive an endowment (Q_t) of perishable goods in each period ($t = 1, 2$), which can be consumed or traded.
- **Perfect Capital Mobility & Interest Rate Parity Condition:** No arbitrage condition implies that the world interest rate (r^*) is equal to the domestic interest rate.

$$r_0 = r_1 = r^\star$$

- **No domestic lending or borrowing**: $B_t = NIIP_t, t \in \{0, 1, 2\}$

3.2 Household Optimization

3.2.1 Budget Constraints

Let C_t be consumption in period t, Q_t be endowment, and B_t be net foreign assets (bonds) held at the end of period t. Initial assets are B_0.

$$\begin{aligned} \text{Period 1:} \quad & C_1 + B_1 = Q_1 + (1 + r_0)B_0 \\ \text{Period 2:} \quad & C_2 + B_2 = Q_2 + (1 + r_1)B_1 \end{aligned}$$

Transversality Condition: $B_2 = 0$ (no debt/assets left at the end), only points on the IBC frontier are feasible.

3.2.2 Intertemporal Budget Constraint (IBC)

Combine the period constraints and $B_2 = 0$ to eliminate B_1:

$$C_1 + \frac{C_2}{1 + r_1} = Q_1 + \frac{Q_2}{1 + r_1} + (1 + r_0)B_0 \equiv \bar{Y}$$

W is lifetime wealth (PV of consumption = PV of endowments + initial wealth). The IBC has a slope of $-(1 + r_1)$ on a (C_1, C_2) graph.

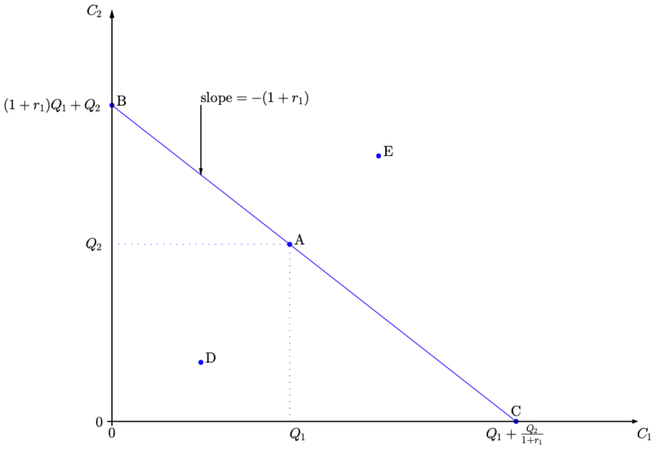


Figure 5: Two-Period Model Basic IBC

3.2.3 Utility Maximization

The household maximizes lifetime utility:

$$U = u(C_1) + \beta u(C_2)$$

where β is the subjective discount factor ($\beta > 0$), $\beta < 1$ is present-biased/ impatient. and $u(\cdot)$ is the period utility function (increasing and concave, $u' > 0, u'' < 0$). The household chooses C_1, C_2 to maximize U subject to the IBC.

3.2.4 Optimality Condition: Consumption Euler Equation

We solve the following (unconstrained) maximization problem as we could substitute $C_2 = (1 + r_1)(\bar{Y} - C_1)$ into the constrained optimization problem.

$$\max_{C_1} U(C_1) + \beta U((1 + r_1)(\bar{Y} - C_1))$$

The solution occurs where the indifference curve is tangent to the IBC.

$$MRS = -\frac{u'(C_1)}{\beta u'(C_2)} = -(1 + r_1)$$

$$u'(C_1) = \beta(1 + r_1)u'(C_2) \quad \text{(Consumption Euler Equation)}$$

3.3 Equilibrium in SOE

An equilibrium is achieved when consumption path (C_1, C_2) satisfies the household's IBC, Consumption Euler Equation and Interest rate parity condition simultaneously, while given exogenous variables $r_0, B_0, Q_1, Q_2, r^{\star}$.

3.4 Current Account Determination

The CA is determined by the difference between output and consumption (plus net investment income).

$$\begin{aligned} CA_1 &= TB_1 + r_0 B_0 = (Q_1 - C_1) + r_0 B_0 \\ TB_1 &= Q_1 - C_1 \\ CA_1 &= B_1 - B_0 \end{aligned}$$

The model implies that households use borrowing and lending (i.e., the current account) to smooth consumption in the face of income fluctuations.

3.4.1 Response to Output Shocks

- **Temporary Negative Shock ($Q_1 \downarrow$, Q_2 unchanged):** Households want to smooth consumption. C_1 falls, but by less than the fall in Q_1. To finance this, they borrow in period 1 ($B_1 < 0$).
  - Initial endowment: A → A'
  - $TB_1 = Q_1 - C_1 < 0$ (Trade Deficit)
  - $CA_1 < 0$ (Current Account Deficit)
  - They must run a trade surplus in period 2 ($TB_2 > 0$) to repay the debt.

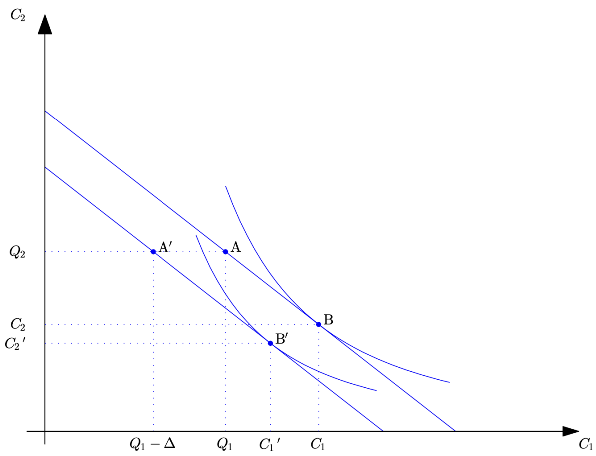


Figure 6: Fall in Period 1 Endowment

- **Permanent Negative Shock ($Q_1 \downarrow$, $Q_2 \downarrow$ by the same amount):** Lifetime wealth shrinks. Households reduce consumption in both periods ($C_1 \downarrow$, $C_2 \downarrow$) by similar amounts.
  - $\Delta TB_1 \approx 0$ (Trade balance changes little)
  - $\Delta CA_1 \approx 0$ (Current account changes little)
  - Adjustment via lower consumption in both periods rather than borrowing that they can't pay back.

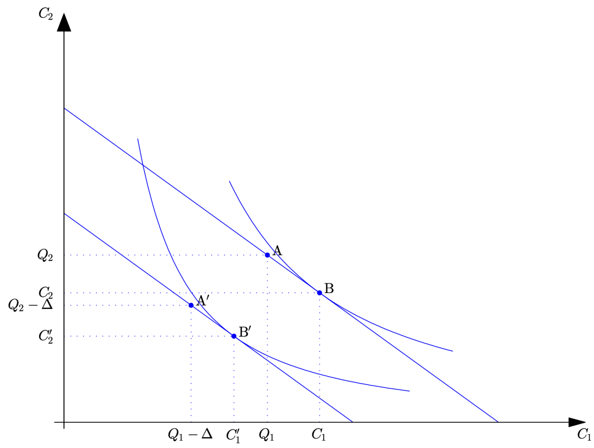


Figure 7: Effect of a Permanent Output Shock.

- **Anticipated Future Income Increase (Q_1 unchanged, $Q_2 \uparrow$ expected):** Lifetime wealth W increases. Households want to start enjoying higher consumption today. C_1 increases through borrowing.
  - $TB_1 = Q_1 - C_1 < 0$ (Trade Deficit)
  - $CA_1 < 0$ (Current Account Deficit)
  - This illustrates CA determination driven by expectations ("good news" about future income).

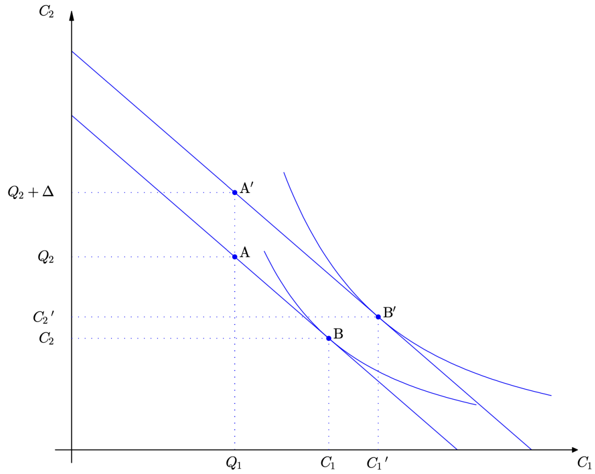


Figure 8: Effect of an Anticipated Future Income Increase.

Key Implication: The CA acts as a buffer, allowing consumption smoothing against temporary or anticipated income shocks. Permanent shocks require adjustment primarily through consumption levels, not borrowing/lending.

4 Chapter 4: Terms of Trade, World Interest Rate, Tariffs

4.1 Terms of Trade

$$TT_1 = \frac{P_1^X}{P_1^M}$$

4.2 IBC

Assume the country exports its endowment good (Q) and imports a consumption good (C). Modify the budget constraints to measure in units of the good (*we could do this because there is only one homogenous good*)

$$\begin{aligned} \text{Period 1:} \quad & C_1 + B_1 = TT_1 Q_1 + (1 + r_0)B_0 \\ \text{Period 2:} \quad & C_2 + B_2 = TT_2 Q_2 + (1 + r_1)B_1 \end{aligned}$$

The IBC becomes:

$$C_1 + \frac{C_2}{1 + r_1} = TT_1 Q_1 + \frac{TT_2 Q_2}{1 + r_1} + (1 + r_0)B_0$$

4.3 Terms of Trade (ToT) Shocks

Implication: A change in the terms of trade (ToT_t) has the same effect on the IBC and optimal consumption as a change in the endowment (Q_t) of the same magnitude. Therefore, the CA response to ToT shocks mirrors the response to endowment shocks:

- **Temporary ToT Deterioration ($ToT_1 \downarrow$, ToT_2 unchanged):** Like a temporary negative income shock. Leads to consumption smoothing by borrowing: $C_1 \downarrow$ but by less than the income loss. Results in CA_1 deficit.
- **Permanent ToT Deterioration ($ToT_1 \downarrow$, $ToT_2 \downarrow$):** Like a permanent negative income shock. C_1 and C_2 decrease significantly. No change in CA balance.
- **Anticipated Future ToT Improvement (ToT_1 unchanged, $ToT_2 \uparrow$ expected):** Like anticipated future income increase. $C_1 \uparrow$ financed by borrowing. Results in CA_1 deficit.

Role of Expectations: The expected duration of a ToT shock is crucial for the CA response. Misjudging the persistence of a shock can lead to outcomes seemingly at odds with the theory ex-post.

4.4 World Interest Rate Shocks (r^*)

An increase in world interest rate from r^* to $r^* + \Delta$, ($\Delta > 0$) leads to:

- **Slope Change:** The slope of the IBC $-(1 + r^*)$ becomes steeper $-(1 + r^* + \Delta)$.
- **Substitution Effect:** Saving becomes more attractive relative to consumption today: $C_1 \downarrow, C_2 \uparrow$.
- **Income Effect:** Depends on the country's initial net asset position (B_0).
  - If Net Debtor ($B_0 < 0$): Lifetime wealth $\bar{Y}$ decreases. $C_1 \downarrow$.
  - If Net Creditor ($B_0 > 0$): Lifetime wealth $\bar{Y}$ increases. $C_1 \uparrow$.

Effect on C_1 and CA_1:

- If Net Debtor: Substitution and Income effects both reduce C_1. $C_1 \downarrow$. Since Q_1 is unchanged, $TB_1 = Q_1 - C_1$ improves. CA_1 improves (becomes less negative or more positive).
- If Net Creditor: Substitution effect reduces C_1, Income effect increases C_1. The net effect on C_1 is ambiguous. Often assumed the substitution effect dominates, so $C_1 \downarrow$ and CA_1 improves.

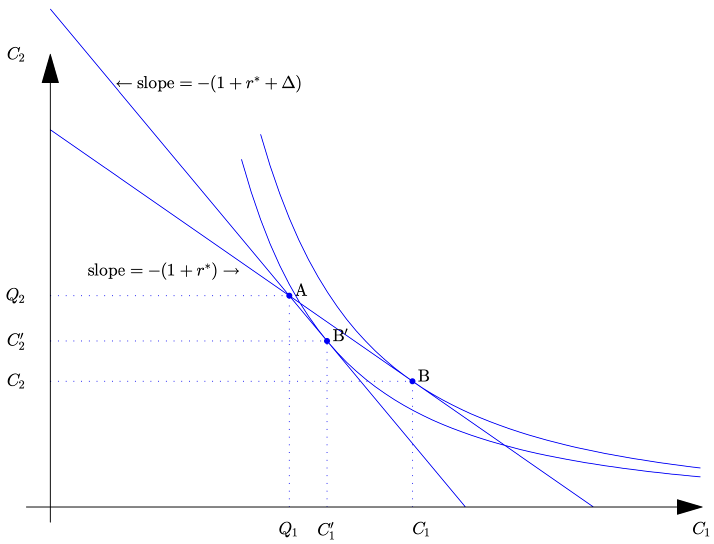


Figure 9: Effect of an Increase in the World Interest Rate.

4.5 Tariffs

Assume the country imports the consumption good (C) and exports the endowment good (Q). The government imposes an import tariff (τ_t) in period t, collected and redistributed lump-sum to households (L_t).

$$\text{Period 1:} \quad (1+\tau_1)C_1 + B_1 = (1+r_0)B_0 + TT_1Q_1 + L_1$$
$$\text{Period 2:} \quad (1+\tau_2)C_2 + B_2 = (1+r_1)B_1 + TT_2Q_2 + L_2$$

The Intertemporal Budget Constraint (IBC) becomes:

$$(1+\tau_1)C_1 + \frac{(1+\tau_2)C_2}{1+r_1} = (1+r_0)B_0 + TT_1Q_1 + \frac{TT_2Q_2}{1+r_1} + +L_1 + \frac{L_2}{1+r_1} = \bar{Y}$$

Government budget constraints:

$$L_1 = \tau_1 C_1$$
$$L_2 = \tau_2 C_2$$

Substitute government constraints into household constraints. The *economy-wide resource constraints* are unchanged by the tariff.

$$C_1 + \frac{C_2}{1+r_1} = TT_1Q_1 + \frac{TT_2Q_2}{1+r_1} + (1+r_0)B_0$$

However, the tariff distorts the relative price of consumption across periods *faced by the household*. The slope faced by the household is $-\frac{(1+r_1)(1+\tau_1)}{(1+\tau_2)}$. The Euler equation becomes:

$$u'(C_1) = \beta(1+r_1)\frac{(1+\tau_1)}{(1+\tau_2)}u'(C_2)$$

4.5.1 Effects of Tariffs

- **Temporary Tariff ($\tau_1 > 0$, $\tau_2 = 0$):** Makes current consumption relatively more expensive for households. Slope of household IBC becomes steeper: $-(1 + r_1)(1 + \tau_1)$. Households substitute away from C_1 towards C_2. $C_1 \downarrow$, $C_2 \uparrow$.
  - $TB_1 = Q_1 - C_1$ improves.
  - CA_1 improves.
  - Welfare reduces as the old BL is no longer achievable
- **Permanent Tariff ($\tau_1 = \tau_2 = \tau > 0$):** The relative price term $\frac{(1+\tau_1)}{(1+\tau_2)} = 1$. The Euler equation is unchanged from the no-tariff case. The slope of the household IBC is $-(1+r^*)$, also unchanged.
  - C_1 and C_2 remain unchanged.
  - TB_1 and CA_1 remain unchanged.
  - A permanent tariff has no effect on the CA in this model.
- **Anticipated Future Tariff ($\tau_1 = 0$, $\tau_2 > 0$ announced):** Makes future consumption relatively more expensive. Slope of household IBC becomes flatter: $-\frac{(1+r^*)}{(1+\tau_2)} < 1 + r^*$. Households substitute towards C_1 away from C_2. $C_1 \uparrow$, $C_2 \downarrow$.
  - $TB_1 = Q_1 - C_1$ deteriorates.
  - CA_1 deteriorates.
  - Also welfare-reducing.

Key Implication: Only temporary or anticipated future tariffs affect the current account by altering the intertemporal relative price of consumption faced by households. Permanent tariffs are neutral in this setup.

5 Chapter 5: CA Determination in a Production Economy

5.1 Model Setup: Introducing Investment

Extends the SOE model by allowing for production using physical capital (I).

- **Production Function:**

$$Q_t = A_t F(I_{t-1}),\ MPK = A_t F'(I_{t-1})$$

  where $A_t > 0$ is productivity (TFP), and I_{t-1} is the physical capital installed at the end of $t-1$. Assume $F' > 0, F'' < 0$ (positive and diminishing marginal product of capital, MPK).

- **Firms:** Choose investment I_1 to maximize profits in period 2, taking r^* as given.

- **Households:** Receive profits (Π) from firms, consume (C), save/borrow (B).

5.2 Firm's Investment Decision

- Period 1: Firms borrow to finance purchase of investment goods

$$D_1^f = I_1$$

- Period 2: Firms produce and sell output, repay principle and interest

$$\Pi_2 = A_2 F(I_1) - (1+r_1)D_1^f = A_2 F(I_1) - (1+r_1)I_1 = Q_2 - (1+r_1)D_1^f$$

Firms maximize Π_2 by choosing I_1. **First-Order Condition (FOC):**

$$\frac{\partial \Pi_2}{\partial I_1} = A_2 F'(I_1) - (1+r_1) = 0$$

$$A_2 F'(I_1) = MPK_2 = 1 + r_1$$

Implication: Firms invest until the expected marginal product of capital in period 2 equals the gross world interest rate. Optimal Investment I_1:

- Decreasing function of the interest rate (r^*). Higher r^* raises the cost of capital, reducing optimal I_1 and profit.

- Increasing function of expected future productivity (A_2). Higher expected A_2 shifts MPK upward, increasing optimal I_1 and profit.

Let the optimal investment function be $I_1^\star = I(r^*, A_2)$.

5.2.1 Profit Function

Profit is the area below MPK and above line $1 + r_1$. Let $I_1^\star$ be the optimal investment level.

- Period 2 Profit Function:

$$\Pi_2 = \int_0^{I_1^\star} A_2 F'(I_1) dI_1 - (1+r_1)I_1^\star$$

- Period 1 Profit Function:

$$\Pi_1 = A_1 F(I_0) - (1+r_0)D_0^f = A_1 F(I_0) - (1+r_0)I_0$$

  where I_0, D_0^f and r_0 are exogenous.

5.3 Household Behavior

Instead of endowment Q_t, households receive profits from firms.

$$Q_1 = \Pi_1(r_0, A_1), Q_2 = \Pi_2(r_1, A_2)$$

Within period budget constraints:

$$\begin{aligned} \text{Period 1:} \quad C_1 + B_1^h &= \Pi_1(r_0, A_1) + (1 + r_0)B_0^h \\ \text{Period 2:} \quad C_2 + B_2^h &= \Pi_2(r_1, A_2) + (1 + r_1)B_1^h \end{aligned}$$

Intertemporal budget constraint (IBC):

$$C_1 + \frac{C_2}{1 + r_1} = \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_1} + (1 + r_0)B_0^h \equiv \bar{Y}$$

which implies $C_2 = (1 + r_1)(\bar{Y} - C_1)$

5.3.1 Optimization of Households

Households maximize lifetime utility:

$$\max_{C_1} U(C_1) + \beta U((1 + r_1)(\bar{Y} - C_1))$$

The solution is the Euler equation (exactly the same as in the endowment economy):

$$u'(C_1) = \beta(1 + r_1)u'(C_2) \quad \text{(Consumption Euler Equation)}$$

5.3.2 Effect of Change in Productivity on Optimal Consumption Path

- **Temporary Increase** in Productivity ($A_1 \uparrow$, A_2 unchanged):
  - $\Pi_1 \uparrow, \Pi_2$ unchanged
  - I_1 stays unchanged since FOC does not depend on A_1.
  - Consumption in both periods increases: $C_1 \uparrow, C_2 \uparrow$.
  - Saving in Period 1 increases to smooth consumption: $S_1 \uparrow$
  - CA balance improves: $CA_1 \uparrow = S_1 \uparrow - I_1$.
- **Anticipated Future Increase** in Productivity ($A_2 \uparrow$ expected):
  - $\Pi_2 \uparrow$ expected, Π_1 unchanged
  - I_1 increases due to higher expected future MPK: $I_1 \uparrow$.
  - Consumption in both periods increases: $C_1 \uparrow, C_2 \uparrow$.
  - Dis-saving in Period 1 to finance higher C_1: $S_1 \downarrow$.
  - CA balance deteriorates: $CA_1 \downarrow = S_1 \downarrow - I_1 \uparrow$.

5.3.3 Effect of Change in Interest Rate on Optimal Consumption Path

Suppose $r_1 \uparrow$, assume $B_0^h = 0$

- Decrease in Period 2 Profit ($\Pi_2 \downarrow$): -ve Income Effect
- Steeper slope of IBC (i.e. Slope is more negative): -ve Income Effect if $B_1^h < 0$
- -ve Substitution Effect: Households substitute away from C_1 towards C_2 by save more & borrow less.
- Overall, $C_1 \downarrow$, $C_2 \uparrow$. Saving in Period 1 increases: $S_1 \uparrow$

5.3.4 Consumption Schedule

Period 1 consumption is decreasing in Period 1 interest rate, increasing in current and future productivity:

$$C_1 = C\big(r_1^-,\, A_1^+,\, A_2^+\big).$$

5.4 National Saving, Investment, and Current Account

5.4.1 National Saving (S_1)

Definition: Difference between GNP and consumption.

$$S_1 = Y_1 - C_1 = r_0 B_0 + A_1 F(I_0) - C_1$$

where $B_0 = B_0^h - D_0^f$, we assume $D_0^f > 0$ (+ve firm borrowing)

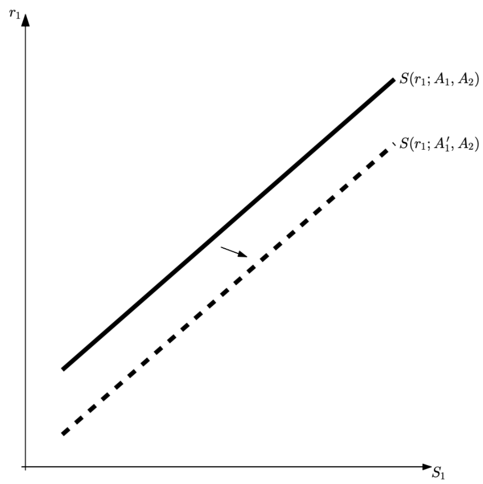
5.4.2 Saving Schedule

r_0, B_0, I_0 are exogenous, only productivity shifts GNP. Hence national saving is increasing in r_1, A_1 (i.e. Higher saving at a given interest rate) while decreasing in A_2.

$$S_1 = Y\big(A_1^+\big) - C\big(r_1^-,\, A_1^+,\, A_2^+\big) = S\big(r_1^+,\, A_1^+,\, A_2^-\big).$$

Shifters:

- **Temporary Productivity Increase ($A_1 \uparrow$):** Increases GNP, $S_1 \uparrow$, saving schedule shifts right.



- **Anticipated Future Productivity Increase ($A_2 \uparrow$):** Decreases GNP, $S_1 \downarrow$, saving schedule shifts left.

5.4.3 CA Determination Schedule

The current account schedule

$$CA_1 = S\big(r_1^+,\, A_1^+,\, A_2^-\big) - I\big(r_1^-,\, A_2^+\big) = CA\big(r_1^+,\, A_1^+,\, A_2^-\big)$$

5.4.4 Metzler Diagram

Shows the CA balance for different levels of the world interest rate r^*.

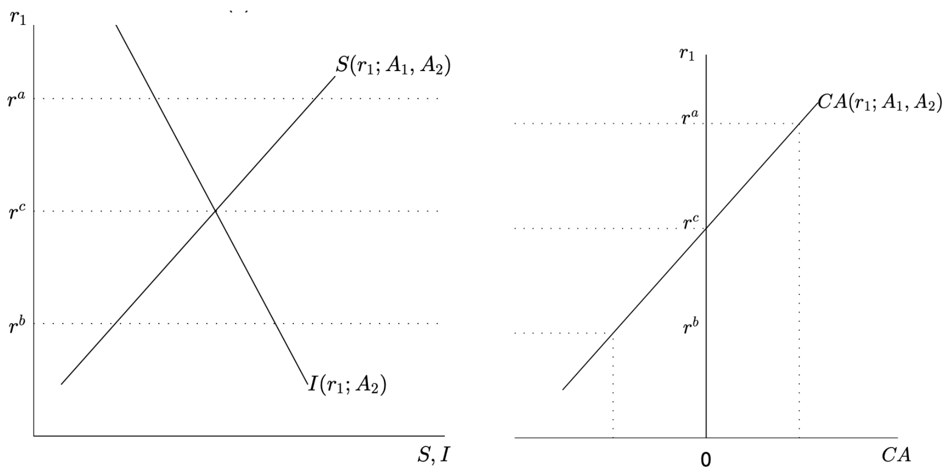
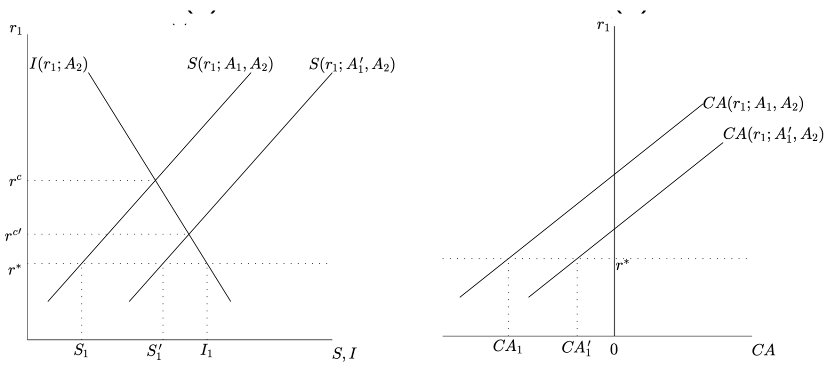
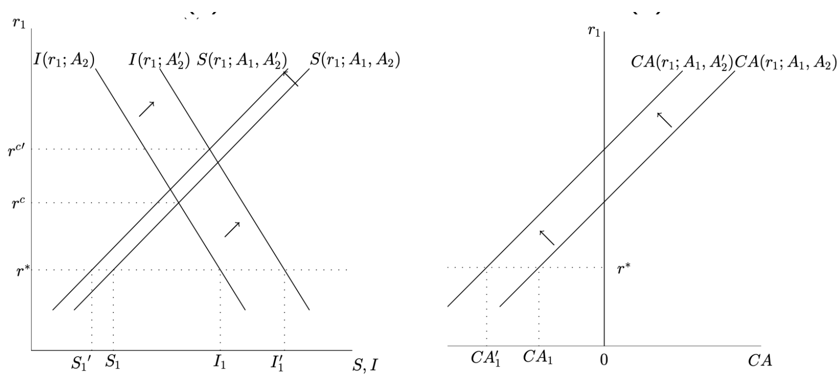


Figure 10: Metzler Diagram: National Saving, Investment, and Current Account Schedules.

- **Slope:** An increase in r^* increases S_1 and decreases I_1. Both effects improve the CA. Thus, the $CA(r^*)$ schedule is upward sloping.
- **Movement along**: A change in world interest rate causes a movement along the CA schedule.
- **Shifts:**
  - **Temporary Productivity Increase** ($A_1 \uparrow$): $S_1 \uparrow$, I_1 unchanged. Saving schedule shifts right & CA schedule shifts right (improves).



  - **Anticipated Future Productivity Increase** ($A_2 \uparrow$): $S_1 \downarrow$ (saving schedule shifts left), $I_1 \uparrow$. Both effects worsen the CA. CA schedule shifts left (deteriorates).



5.5 Summary

	$r^* \uparrow$		$A_1 \uparrow$		$A_2 \uparrow$	
	Open	Closed	Open	Closed	Open	Closed
S_1	$\uparrow$	–	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow^*$
I_1	$\downarrow$	–	–	$\uparrow$	$\uparrow$	$\uparrow^*$
CA_1	$\uparrow$	–	$\uparrow$	–	$\downarrow$	–
r_1	$\uparrow$	–	–	$\downarrow$	–	$\uparrow$

5.6 Application: Terms of Trade Shocks in a Production Economy

Suppose the country produces oil for export and consumes the only import good (e.g. food). Also physical capital is imported. Relative price of capital to food is unity.

5.6.1 Level of Profit

The ToT term always enters the profit function as a multiplicative factor before the production function:

- Period 1: $\Pi_1 = TT_1 A_1 F(I_0) - (1 + r_0)I_0$
- Period 2: $\Pi_2 = TT_2 A_2 F(I_1) - (1 + r_1)I_1$

5.6.2 Profit Maximization of Firms

Firms maximize profit by choosing I_1 to maximize Π_2:

$$\frac{\partial \Pi_2}{\partial I_1} = TT_2 A_2 F'(I_1) - (1 + r_1) = 0$$

5.6.3 Investment Schedule

$$I_1 = I(\overset{-}{r_1}; TT_2 \overset{+}{A_2})$$

5.7 Saving & CA Schedules

- **GNP**: $Y_1 = r_0 B_0 + Q_1 = r_0 B_0 + (1 + r_0)I_0$
- **Saving Schedule**: $S_1 = S(r_1^+, TT_1 A_1^+, TT_2 A_2^-)$
- **Current Account Schedule**: $CA_1 = CA(r_1^+, TT_1 A_1^+, TT_2 A_2^-)$

Notice that the implication of shock is the same, all you need is to replace A_1 by $TT_1 A_1$ and A_2 by $TT_2 A_2$ in the previous section.

6 Chapter 6: Uncertainty and the Current Account

6.1 Results

Under uncertainty (compared to perfect foresight):

- Risk-averse households engage in precautionary saving in Period 1 ($S_1 \uparrow, C_1 \downarrow$)
- Given constant Q_1, trade balance MUST improve ($TB_1 \uparrow$)
- Current account balance MUST improve ($CA_1 \uparrow$)

6.2 Modelling: Endowment Economy with Uncertainty

Modify the Chapter 3 model: Suppose $Q_1 = Q$ and Q_2 becomes stochastic, we model a mean-preserving increase in uncertainty:

$$Q_2 = \begin{cases} Q+\sigma, & \Pr(Q+\sigma) = \frac{1}{2}, \\ Q-\sigma, & \Pr(Q-\sigma) = \frac{1}{2}; \end{cases} \tag{1}$$

$$\mathbb{E}[Q_2] = \tfrac{1}{2}(Q+\sigma) \ + \ \tfrac{1}{2}(Q-\sigma) \ = \ Q, \tag{2}$$

$$\text{Var}(Q_2) = \mathbb{E}\big[(Q_2 - \mathbb{E}[Q_2])^2\big] = \tfrac{1}{2}(Q+\sigma-Q)^2 \ + \ \tfrac{1}{2}(Q-\sigma-Q)^2 = \sigma^2. \tag{3}$$

6.3 State-Contingent Inter-temporal Budget Constraints

We assume $B_0 = 0, r^\star = 0$, $\beta = 0$ and log additive preferences.

$$C_1 + C_2 = \begin{cases} Q + \big(Q+\sigma\big), & \text{with probability } \frac{1}{2} \text{ [Good State]}, \\ Q + \big(Q-\sigma\big), & \text{with probability } \frac{1}{2} \text{ [Bad State]}. \end{cases}$$

$$C_2 = \begin{cases} 2Q+\sigma-C_1, & \text{with probability } \frac{1}{2}, \\ 2Q-\sigma-C_1, & \text{with probability } \frac{1}{2}. \end{cases}$$

6.4 Incomplete Markets: Precautionary Saving

Assume only a single risk-free bond exists, paying $(1+r^*)$ in period 2 regardless of the state. Markets are incomplete. Household maximizes expected utility:

$$\max_{C_1,B_1} \quad u(C_1)+\beta E[u(C_2)] = \log(C_1)+\frac{1}{2}\log(C_2^H)+\frac{1}{2}\log(C_2^L) = \log(C_1)+\frac{1}{2}\log(2Q+\sigma-C_1)+\frac{1}{2}\log(2Q-\sigma-C_1)$$

The F.O.C. is:

$$\frac{1}{C_1} = \frac{1}{2}\left[\frac{1}{2Q+\sigma-C_1} + \frac{1}{2Q-\sigma-C_1}\right]$$

We evaluate both sides of the F.O.C. at $C_1 = Q$ (i.e. the optimum under certainty), notice that

$$\text{RHS} = \frac{1}{2}\Big[\frac{1}{2Q+\sigma-Q} \ + \ \frac{1}{2Q-\sigma-Q}\Big] = \frac{Q}{Q^2-\sigma^2} = \frac{1}{Q}\Big(\frac{Q^2}{Q^2-\sigma^2}\Big) > \frac{1}{Q} = \text{LHS}\,.$$

Given that LHS is decreasing in C_1 and RHS is increasing in C_1, the F.O.C. implies that the optimal C_1 must be less than Q:

$$C_1 < Q \quad (\text{i.e. } C_1 = Q - S_1)$$

Implication: Uncertainty induces households to reduce current consumption (C_1) and increase saving ($S_1 = Q_1 - C_1$). This is called **precautionary saving**.

- Higher uncertainty ($\sigma \uparrow$) leads to lower C_1 and higher S_1.
- Higher uncertainty ($\sigma \uparrow$) leads to improvements in NIIP ($B_1 \uparrow = Q - C_1 \downarrow$)
- Higher uncertainty ($\sigma \uparrow$) leads to **improvements** in TB and CA. Vice versa.

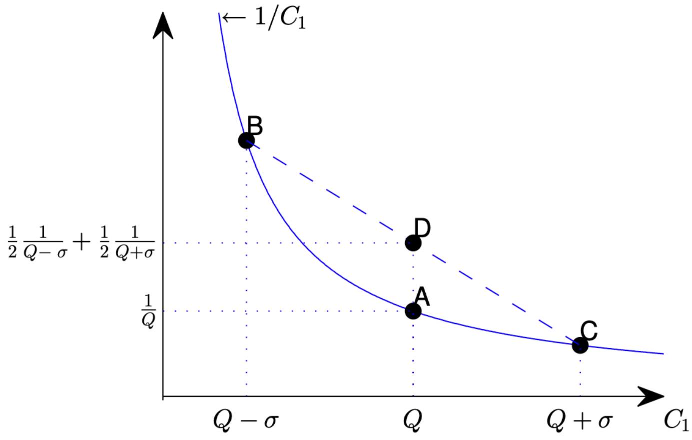


Figure 11: Precautionary Saving due to Convex Marginal Utility.

6.5 Complete Markets: Risk Sharing

6.5.1 Arrow-Debreu (AD) Securities

Suppose we have two AD securities and market is complete.

- **State H Security (AD^H):** Pays 1 in state H, 0 in state L. State price is p^H.
- **State L Security (AD^L):** Pays 1 in state L, 0 in state H. State price is p^L.

Let B^H and B^L be the quantities of AD securities purchased in period 1.

6.5.2 Optimization under Complete Markets

Period 1 Budget Constraint:

$$C_1 + p^H B^H + p^L B^L = Q_1$$

Period 2 Budget Constraints:

$$\begin{aligned} \text{State H:} \quad C_2^H &= Q + \sigma + B^H \\ \text{State L:} \quad C_2^L &= Q - \sigma + B^L \end{aligned}$$

Households maximize expected utility:

$$\max_{B^H, B^L} \left\{ \log(Q - P^H B^H - P^L B^L) \ + \ \tfrac{1}{2} \log(Q + \sigma + B^H) \ + \ \tfrac{1}{2} \log(Q - \sigma + B^L) \right\}$$

Optimality Conditions (FOCs):

$$\frac{1}{C_1} = \frac{1}{p^H} \cdot \frac{1}{2} \cdot \frac{1}{C_2^H}$$
$$\frac{1}{C_2} = \frac{1}{p^L} \cdot \frac{1}{2} \cdot \frac{1}{C_2^L}$$

Free Capital Mobility Assumption: The state prices p^H and p^L are determined by the world interest rate r^*, such that:

$$p^H = p^{H\star},\ p^L = p^{L\star}$$

The world interest rate is

$$1 + r^* = \frac{1}{p^{H\star} + p^{L\star}}$$

6.5.3 No Arbitrage Pricing

Under no-arbitrage, foreign investors make zero expected profits. Suppose they reinvest the earning from selling AD securities in Period 1 into a risk-free bond paying $1 + r^*$ in Period 2. Let $r^\star = 0$, then the state prices are:

$$p^H = p^L = \frac{1}{2}$$

6.5.4 Equilibrium

Given the state prices, the optimality conditions & budget constraints imply

$$C_1 = C_2^H = C_2^L = Q$$

. Hence, we have shown that in complete market, households smooth consumption by creating an insurance themselves.

- Short σ amount of B^H; Long σ amount of B^L
- Trade balance and current account are zero.
- Net Asset Position = 0.

Reality Check: Real-world financial markets are incomplete; perfect state-contingent securities for all risks do not exist. Therefore, the precautionary saving model (incomplete markets) is likely more relevant.

7 Chapter 8: Twin Deficits - Fiscal Deficits and the Current Account

7.1 Opposing Views on Tax Cut

- **Ricardian Equivalence:** Tax cuts do not affect the current account. Households save the tax cut, offsetting the fiscal deficit.
- **Twin Deficits View:** Tax cuts worsen the current account by increasing the fiscal deficit, leading to higher borrowing and lower national saving.

7.2 Modelling - Government Sector

We extend the production economy in Chapter 5 to include government spending and taxes. Suppose the following:

- Government spends G_t. [Exogenous]
- Government collects lump-sum taxes T_t.
- Government holds bonds B^g_{t-1} at the start of Period t. B^g_0 is exogenous.

7.2.1 Government Budget Constraints

- Period 1: $G_1 + B^g_1 = T_1 + (1 + r_0)B^g_0$
- Period 2: $G_2 + B^g_2 = T_2 + (1 + r_1)B^g_1$

Apply the Transversality Condition $B^g_2 = 0$, rearrange to get the gov't IBC.

$$G_1 + \frac{G_2}{1 + r} = (1 + r_0)B^g_0 + T_1 + \frac{T_2}{1 + r}$$

Ricardian Equivalence Proposition: Since G_1, G_2, B^g_0 are all exogenous, a tax cut in Period 1 MUST be compensated by a tax increase in Period 2, such that the gov't IBC holds.

7.3 Modelling - Private Sector

7.3.1 Firms

Given that firm's optimal investment decision is independent of taxation, Period 2 profit is $\Pi(r_1)$.We also assume income in Period 1 as Q_1 rather than $A_1F(I_0)$. All derivations remain the same as in Chapter 5.

7.3.2 Households

Suppose households maximize log additive utility with $\beta = 0$: $U = \log(C_1) + \log(C_2)$.

Budget constraints:

$$\begin{aligned} \text{Period 1:} \quad & C_1 + B^h_1 = Q_1 + (1 + r_0)B^h_0 - T_1 \\ \text{Period 2:} \quad & C_2 + B^h_2 = \Pi(R_1) + (1 + r_1)B^h_1 - T_2 \end{aligned}$$

Intertemporal budget constraint (IBC):

$$C_1 + \frac{C_2}{1 + r_1} = Q_1 + \frac{\Pi(r_1)}{1 + r_1} + (1 + r_0)B^h_0 - T_1 - \frac{T_2}{1 + r_1} \equiv \bar{Y}$$

Rearranging the IBC gives:

$$C_2 = (1 + r_1)\left(\bar{Y} - C_1\right) = (1 + r_1)[(1 + r_0)B^h_0 + Q_1 - T_1 - C_1] + \Pi(r_1) - T_2$$

7.3.3 Household Optimization

Optimal consumption path is:

$$c_1 = \tfrac{1}{2}\Big((1+r_0)B_0^h \ + \ Q_1 - T_1 \ + \ \tfrac{\Pi(r_1)-T_2}{1+r_1}\Big),$$
$$c_2 = \tfrac{1+r_1}{2}\Big((1+r_0)B_0^h \ + \ Q_1 - T_1 \ + \ \tfrac{\Pi(r_1)-T_2}{1+r_1}\Big).$$

Note that C_1 and C_2 depends only on the PV of taxes.

7.4 Equilibrium

Initial NIIP: $B_0 = B_0^h + B_0^g$. We combine gov't budget constraint, interest rate parity condition and household optimization to derive the equilibrium conditions.

$$C_1 = \frac{1}{2}\Big((1+r_0)\,B_0 + Q_1 - G_1 + \frac{\Pi(r^*) - G_2}{1+r^*}\Big), \tag{4}$$

$$C_2 = \frac{1+r^*}{2}\Big((1+r_0)\,B_0 + Q_1 - G_1 + \frac{\Pi(r^*) - G_2}{1+r^*}\Big), \tag{5}$$

$$\mathrm{TB}_1 = Q_1 - C_1 - G_1 - I(r^*) = \frac{1}{2}\Big(-(1+r_0)\,B_0 + Q_1 - G_1 + \frac{\Pi(r^*) - G_2}{1+r^*}\Big) \ - \ I(r^*), \tag{6}$$

$$\mathrm{CA}_1 = \mathrm{TB}_1 + r_0\,B_0 = \frac{1}{2}\Big(-(1-r_0)\,B_0 + Q_1 - G_1 + \frac{\Pi(r^*) - G_2}{1+r^*}\Big) \ - \ I(r^*). \tag{7}$$

Conclusion - Ricardian Equivalence: Notice that the equilibrium conditions DO NOT depend on the tax level T_1 or T_2. Therefore, households necessarily save the entire tax cut given that gov't spending stay unchanged, leaving consumption and current account unchanged. As a result, tax cut **DOES NOT** cause twin deficit.

7.5 Ricardian Equivalence

7.5.1 Gov't saving

$$S_1^g = r_0 B_0^g + T_1 - G_1$$

Given that $G_1, r_0 B_0^g$ are exogenous, any change in T_1 must satisfies

$$\Delta S_1^g = \Delta T_1$$

7.5.2 Private saving

$$S_1^p = Q_1 + r_0 B_0^h - C_1 - T_1$$

Given that Q_1 and $r_0 B_0^h$ are exogenous and C_1 depends only on the total NPV of taxes (which did not change), any change in T_1 must satisfy

$$\Delta S_1^p = -\Delta T_1$$

7.5.3 National saving

$$S_1 = S_1^p + S_1^g$$

Any change in national saving (for a given level of gov't spending) must satisfy:

$$\Delta S_1 = \Delta S_1^p + \Delta S_1^g = -\Delta T_1 + \Delta T_1 = 0$$

Given that national saving does not change + Investment stays fixed, the current account must also remain unchanged:

$$\Delta CA_1 = \Delta S_1 - \Delta I = 0$$

7.6 Government Spending & Twin Deficits

Intuition: A change in G_1 (that can be financed by an increase in taxes today or by an increase in debt), has the SAME effect as a change in the endowment in the opposite direction. (i.e. $G_1 \uparrow$ is like $Q_1 \downarrow$ and $G_2 \downarrow$ is like $\Pi(r^\star) \uparrow$)

7.6.1 Temporary Increase in G_1 (G_2 unchanged)

- Households: Partial crowding-out of private consumption. $C_1 \downarrow$ (due to lower lifetime wealth).
$$\Delta C_1 = -\frac{1}{2}\Delta G_1$$
- Firms: No effect on investment as r_1 is exogenously given. $\Delta I_1 = 0$
- Domestic Absorption > 0:
$$\Delta A_t = \Delta(C_1 + I_1 + G_1) = -\frac{1}{2}\Delta G_1 + 0 + \Delta G_1 = \frac{1}{2}\Delta G_1 > 0$$
- Trade Balance:
$$\Delta TB_1 = \Delta(Q_1 - C_1 - G_1 - I_1) = \Delta Q_1 - \frac{1}{2}\Delta G_1 - \Delta G_1 - 0 = -\frac{1}{2}\Delta G_1 < 0$$
- CA Balance worsens! $\Delta CA_1 = \Delta TB_1 < 0$

Therefore, a **temporary increase in G_1 causes twin deficits:** (if not finance by a tax hike)

- Fiscal Deficit: $S_1^g \downarrow$ (i.e. $G_1 \uparrow$)
- Current Account Deficit: $CA_1 \downarrow$ (i.e. $TB_1 \downarrow$)

7.6.2 Expected Future Increase in G_2 (G_1 unchanged)

- Households: Reduce C_1 to smooth consumption
$$\Delta C_1 = -\frac{1}{2} \cdot \frac{1}{1 + r^\star}\Delta G_2 < 0$$
- TB & CA: Notice that Q_1, G_1 and $r^\star$ remain unchanged,
$$\Delta TB_1 = \frac{1}{2} \cdot \frac{1}{1 + r^\star}\Delta G_2 > 0$$
and
$$\Delta CA_1 = \Delta TB_1 > 0$$

Therefore, an **anticipated increase in gov't spending** leads to an improvement in TB and CA in Period 1; We do not observe twin deficits in this case.

7.6.3 Permanent Increase in G_1 and G_2

- Households: Reduce consumption in both periods by $\Delta G_1 = \Delta G_2 = \Delta G$
- Current Account: Little change
$$\Delta CA_1 = -\frac{r^\star}{2(1 + r^\star)}\Delta G$$
- Permanent spending hike does NOT induce twin deficits.

7.7 Failures of Ricardian Equivalence

Ricardian Equivalence is a fragile result and would fail if any of the following assumptions is relaxed:

- **No borrowing constraint:** Households smooth consumption according to PIH.
- **Expect tax cut to be followed by tax hike**
- **Taxes are lump sum**: No distortion to relative price & incentives

7.7.1 Borrowing constraints

Suppose borrowing constraint $B_1^h \geq 0$ (i.e. saving-only) is binding for all tax rate. Therefore, households are **Hand-To-Mouth** and choose that is lower than the unconstrained level.

$$C_1^{\text{constrained}} = C_1 - T_1 < C_1^\star$$

Effect of Tax Cut ($\Delta T_1 < 0$):

- HTM Behavior (MPC = 1): $\Delta C_1 = -\Delta T_1 \implies$ Failure of Ricardian Equivalence.
- G_1 and I_1 stays unchanged as they are not constrained.
- TB_1 and CA_1 worsen:

$$\Delta TB_1 = \Delta Q_1 - \Delta C_1 - \Delta G_1 - \Delta I_1 = 0 - (-\Delta T_1) - 0 - 0 = \Delta T_1 < 0$$

- Tax Cut cause public dis-saving not matched by private saving.

Therefore, **tax cut causes twin deficits** under borrowing Constraints.

7.7.2 Intergenerational Effects

- Current generation who dies tomorrow benefits from tax cut but does not pay for it.

$$\Delta C_1 = -\Delta T_1 > 0$$

- Neither gov't nor firms are short-lived, so G_1, I_1 remain unchanged.
- Trade Balance and Current Account worsen

Therefore, **tax cut causes twin deficits** under intergenerational effects.

7.7.3 Distortionary Taxes

Intuition: Households facing VAT (τ_1, τ_2) spend more in t_1 and less in t_2 as VAT distorts relative prices.

- Households: With-in period budget constraints:

$$(1+\tau_1)C_1 + B_1^h = Q_1 + (1+r_0)B_0^h$$
$$(1+\tau_2)C_2 + B_2^h = \Pi(r_1) + (1+r_1)B_1^h$$

  Intertemporal budget constraint:

$$(1+\tau_1)C_1 + \frac{(1+\tau_2)C_2}{1+r_1} = Q_1 + \frac{\Pi(r_1)}{1+r_1} + (1+r_0)B_0^h$$

Assume log additive utility, the Euler equation is:

$$\frac{C_2}{C_1} = \frac{1+\tau_1}{1+\tau_2} \cdot (1+r_1)$$

Notice that cutting Period 1 taxes ($\tau_1 \downarrow$) and hike back Period 2 taxes ($\tau_2 \uparrow$) leads to a higher consumption in Period 1 ($C_1 \uparrow$) and lower consumption in Period 2 ($C_2 \downarrow$).

- Firms: Firms' investment decision is independent of taxation, so I_1 remains unchanged.
- Equilibrium:

$$C_1 = \frac{1}{2(1+\tau_1)}\Big((1+r_0)B_0^h + Q_1 + \frac{\Pi(r_1)}{1+r_1}\Big)$$

Notice that C_1 is strictly decreasing in τ_1. We will have twin deficits.

- Gov't: VAT also leads to fiscal deficit.

7.8 Imperfect Capital Mobility

7.8.1 Modelling Imperfect Capital Mobility

Assumptions:

- Net external creditor ($B_1 < 0$): $r_1 = r^\star$
- Net external debtor ($B_1 > 0$): $r_1 > r^\star$ and is increasing function of B_1. (*Reflects the risk premium*)
- $B_0 = 0 \implies B_1 = CA_1$

Imperfect Capital Mobility:

$$r = \rho(B_1) = \rho(-CA_1) = \begin{cases} r^*, & CA_1 \geq 0, \\ > r^* \text{ with } \rho' > 0, & CA_1 < 0. \end{cases}$$

Notice that CA deficit pushes up interest rate, which in turn reduces investment and consumption.

7.9 Modelling VAT under Imperfect Capital Mobility

$$\mathrm{CA}_1 = r_0\, B_0 + Q_1 - \frac{1}{2\big(1+\tau_1\big)}\Big(\left(1+r_0\right) B_0^h + Q_1 + \frac{\Pi(r^*)}{1+r^*}\Big) - I(r_1) - G_1.$$

Hence, CA_1 is a function of:

$$CA_1 = CA(\overset{+}{r_1}, \overset{+}{Q_1}, \overset{+}{\tau_1}, \overset{-}{G_1})$$

7.9.1 Comparison: Autarky, Perfect and Imperfect Capital Mobility

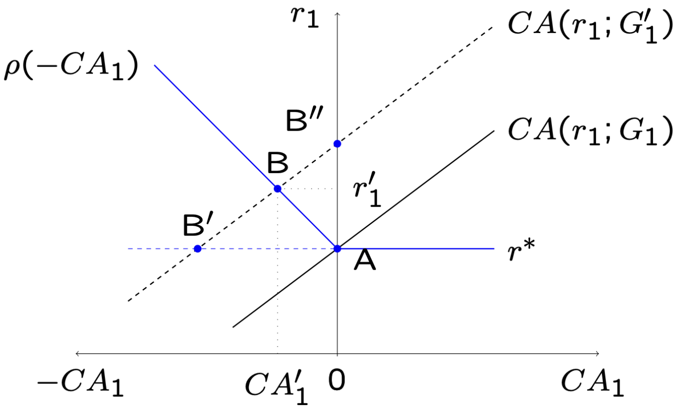


Figure 12: Fiscal Policy under Imperfect Capital Mobility.

8 Chapter 9: Real Exchange Rate and Purchasing Power Parity (PPP)

8.1 Nominal Exchange Rate

Definition: The number of domestic currency that trade against each unit of foreign currency.

$$\boxed{\epsilon = \frac{\text{Home Currency}}{\text{Foreign Currency}}}$$

A rise in ϵ signals a depreciation of the domestic currency.

8.2 Law of One Price (LOOP)

The LOOP holds when a good costs the same abroad and at home

$$P = \epsilon P^{\star}$$

where P is the home price, $P^{\star}$ is the foreign price, and ϵ is the nominal exchange rate. Assumes identical and *perfectly tradable* goods, no transport or distribution costs; holds mainly for commodities (e.g., oil, gold, agricultural, metal...).

8.3 Real Exchange Rate (RER)

$$\boxed{e = \epsilon \times \frac{P^{\star}}{P}}$$

Interprets relative price levels of representative baskets.

8.3.1 Big-Mac Index

How many U.S. Big Macs it takes to buy one Big Mac abroad?

$$e_{\text{BM}} = \frac{\epsilon\, P^{\star}_{\text{BM}}}{P_{\text{BM}}}$$

The LOOP holds when $e = 1$. Empirically, it doesn't hold because non-tradable cost of productions dominate cost spectrum of Big Mac (labor, utility bills, rent...).

8.4 Purchasing Power parity (PPP)

8.4.1 Absolute PPP

Definition: The absolute PPP holds when the RER equals 1, meaning that the price of a basket of goods is the same in both countries when expressed in a common currency. (i.e. Generalization of LOOP)

8.4.2 PPP exchange rate

Definition: Nominal exchange rate that would make the consumption basket in two countries equally expensive (Hence PPP holds):

$$\epsilon_{\text{PPP}} P^{\star} = P \quad \Longrightarrow \quad RER = \frac{\epsilon_{\text{PPP}} P^{\star}}{P} = 1$$

8.4.3 Valuation of Currency

- $\epsilon_{\text{PPP}} > \epsilon$: Home is more expensive than abroad ($P > \epsilon P^\star$), home currency is overvalued.
- $\epsilon_{\text{PPP}} < \epsilon$: Home is cheaper than abroad ($P < \epsilon P^\star$), home currency is undervalued.

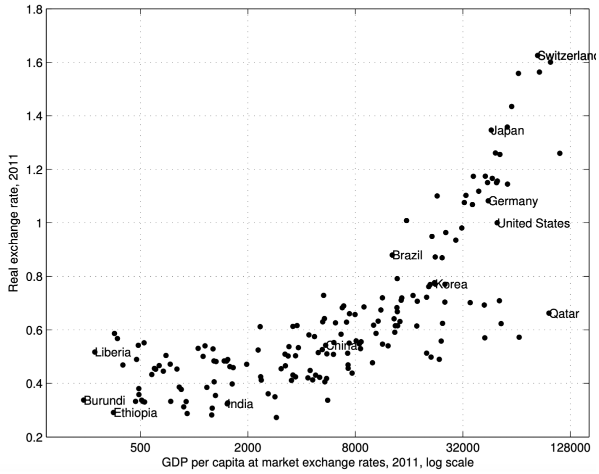
8.4.4 Living Standard Comparison

It's flawed to compare GDP using ϵ because it does not account for price level differences. Instead, use PPP exchange rate:
Per Capita GDP at PPP Exchange Rate:

$$R = \frac{\text{GDP}^{US}/P^{US}}{\text{GDP}^\star/P^\star} = \frac{1}{P^{US}/P^\star} \times \frac{GDP^{US}}{GDP^\star} = \frac{1}{\epsilon_{PPP,\star}} \times \frac{GDP^{US}}{GDP^\star}$$

8.4.5 Balassa-Samuelson Effect

Rich countries (i.e. countries with higher per capita income) tend to have higher prices.



Higher price level in advanced economies could be the result of higher productivity in tradable sectors experience higher wages, which give rise to higher prices of non-tradable goods in general.

8.5 Relative PPP

Relative version uses price *changes*, it holds when equal to naught:

$$\Delta e_t \equiv \frac{\Delta \epsilon_t P_t^\star}{P_t} = 0$$

First-order Taylor approximation of RER growth rate yields:

$$\boxed{\varepsilon_t^r = \varepsilon_t + \pi_t^{US} - \pi_t}$$

where ε_t^r is the real depreciation of a country's currency against USD (i.e. Growth rate of RER). Relative PPP holds when

$$\varepsilon_t = \pi_t - \pi_t^{US}$$

Empirical Evidence: Many countries satisfy relative PPP in the long-run, but not in the short-run.

8.6 Border Effect (Engel & Rogers, 1996)

Real Exchange Rate Between Cities

For any basket g and cities c_1, c_2 at time t, the bilateral real exchange rate is

$$e^g_{c_1,c_2,t} \;=\; \frac{\epsilon_{c_1,c_2,t}\, P^g_{c_2,t}}{P^g_{c_1,t}},$$

where

- $P^g_{c_i,t}$ is the local price index of basket g in city c_i,
- $\epsilon_{c_1,c_2,t}$ is the nominal *city–city* exchange rate (if c_1, c_2 lie in one country, $\epsilon = 1$).

Define

$$\sigma^g_{c_1,c_2} \;=\; \text{stdev}_t\big(\Delta \ln e^g_{c_1,c_2,t}\big),$$

so that larger σ indicates greater short-run deviations from relative PPP.

Econometric Specification

Engel and Rogers estimate the relationship

$$\sigma^g_{c_1,c_2} = \;\beta_0 \;+\; \beta_1 \ln d_{c_1,c_2} \;+\; \beta_2\, B_{c_1,c_2} \;+\; u^g_{c_1,c_2},$$

with

$$\beta_1 = 0.00106, \quad \beta_2 = 0.0119,$$

where

- d_{c_1,c_2} is the great–circle distance (miles) between c_1, c_2,
- $B_{c_1,c_2} = 1$ if an international border separates them, 0 otherwise,

Furthermore, existence of boarder is equivalent to increasing distance by 12,000 miles.

Economic Interpretation

- *Transportation costs*: larger price differentials are needed to induce cross-city shopping over greater distances.
- *Nominal rigidities*: local-currency prices stick, so exchange-rate swings induce cross-border RER movements absent within-country offsets.
- *Trade frictions*: tariffs, quotas and regulations further segment cross-border markets.

8.7 Non-Tradables and the RER

Let price levels to be some average of prices of tradables and non-tradables:

$$P = \phi(P_T, P_N), \qquad P^\star = \phi(P^\star_T, P^\star_N)$$

where

- P_T, P_N = Price of tradables and non-tradables at home,
- $P^\star_T, P^\star_N$ = Price of tradables and non-tradables abroad,
- $\phi(\cdot)$ is increasing in both arguments and homogeneous of degree 1

LOOP holds for tradables but not non-tradables:

$$P_T = \epsilon P^\star_T, \qquad P_N \neq \epsilon P^\star_N$$

8.7.1 Real Exchange Rate with Non-Tradables

Using homogeneity for both tradables and non-tradables, we can express the RER as:

$$e = \frac{\epsilon P^\star}{P} = \epsilon \frac{\phi(P_T^\star, P_N^\star)}{\phi(P_T, P_N)} = \epsilon \frac{P_T^\star \phi(1, P_N^\star/P_T^\star)}{P_T \phi(1, P_N/P_T)}$$

Using LOOP for tradables:

$$e = \frac{\phi\big(1, \frac{P_N^\star}{P_T^\star}\big)}{\phi\big(1, \frac{P_N}{P_T}\big)}$$

Hence $e < 1$ when non-tradables are relatively cheaper abroad.

8.8 Trade Barriers

Modelling the Trade Barrier

Let

$$P_M^\star = \text{world price of importable goods}, \quad P_X^\star = \text{world price of exportable goods},$$

$$P_M,\, P_X = \text{domestic prices of importable and exportable goods},$$

and let the nominal exchange rate be ε. Suppose the Law of One Price holds for each type of good:

$$P_X = \varepsilon\, P_X^\star, \qquad P_M = \varepsilon\, P_M^\star.$$

Define aggregate price levels by a price aggregator

$$P = \phi(P_X, P_M), \qquad P^\star = \phi(P_X^\star, P_M^\star).$$

Without Trade Barrier

When all goods are tradable and no barrier exists,

$$e \;=\; \frac{\varepsilon\, P^\star}{P} \;=\; \varepsilon\, \frac{\phi(P_X^\star, P_M^\star)}{\phi(P_X, P_M)} \;=\; \varepsilon\, \frac{\varepsilon^{-1}\phi(\,\varepsilon P_X,\ \varepsilon P_M\,)}{\phi(P_X, P_M)} \;=\; \frac{\phi(\varepsilon P_X^\star,\, \varepsilon P_M^\star)}{\phi(P_X, P_M)}.$$

By homogeneity of ϕ and LOOP compatibility, $\phi(\varepsilon P_X^\star, \varepsilon P_M^\star) = \phi(P_X, P_M)$, hence

$$e = \frac{\phi(P_X, P_M)}{\phi(P_X, P_M)} = 1.$$

With Trade Barrier (Import Tariff τ)

Suppose the domestic government imposes a tariff τ on imports. Then importers pay $\varepsilon P_M^\star$ to the foreign producer plus $\tau \varepsilon P_M^\star$ to the government, so

$$P_M = (1+\tau)\, \varepsilon\, P_M^\star, \qquad P_X = \varepsilon\, P_X^\star.$$

The real exchange rate becomes

$$e = \frac{\varepsilon\, \phi(P_X^\star, P_M^\star)}{\phi(P_X, P_M)} = \varepsilon\, \frac{\phi\big(P_X^\star,\ P_M^\star\big)}{\phi\big(\varepsilon P_X^\star,\ (1+\tau)\varepsilon P_M^\star\big)} = \frac{\phi\big(P_X^\star,\ P_M^\star\big)}{\phi\big(P_X^\star,\ (1+\tau) P_M^\star\big)} \;<\; 1.$$

Thus, an import tariff makes the domestic country relatively more expensive.

8.9 Home Bias

Definition: A preference for domestically produced goods CPI constructions: Suppose b refers to the price of beef and c refers to the price of cars, then the price index is given by a Cobb-Douglas aggregator:

$$P = (P_b)^{\gamma}(P_c)^{1-\gamma}, \qquad P^{\star} = (P_b^{\star})^{\gamma^{\star}}(P_c^{\star})^{1-\gamma^{\star}}$$

If LOOP holds for both goods, define $\epsilon = \text{EUR/ARG}$ then

$$e = \frac{\epsilon P^{\star}}{P} = \frac{\epsilon (P_b^{\star})^{\gamma^{\star}}(P_c^{\star})^{1-\gamma^{\star}}}{(P_b)^{\gamma}(P_c)^{1-\gamma}} = \frac{\epsilon \left(\frac{P_b}{\epsilon}\right)^{\gamma^{\star}}\left(\frac{P_c}{\epsilon}\right)^{1-\gamma^{\star}}}{(P_b)^{\gamma}(P_c)^{1-\gamma}} = \left(\frac{P_c}{P_b}\right)^{\gamma-\gamma^{\star}} \neq 1.$$

Notice that the deviation from PPP is driven by home bias ($\gamma \neq \gamma^{\star}$). Suppose domestic economy favors beef over cars ($\gamma > \gamma^{\star}$), then $P_b \uparrow$ causes $e \downarrow$ (i.e. real appreciation of peso).

8.10 Price Index Micro-foundation & Welfare

(Cobb-Douglas) Consumption aggregator:

$$C = C_T^{\gamma} C_N^{1-\gamma}$$

Consumer price level from expenditure minimization:

$$P = \min_{C_T, C_N} \big(P_T C_T + P_N C_N\big) \quad \text{subject to} \quad C_T^{\gamma} C_N^{1-\gamma} = 1$$

Solve the constrained minimization problem, then eliminate C_T, C_N from the object function:

$$C_T = \left(\frac{\gamma}{1-\gamma} \cdot \frac{P_N}{P_T}\right)^{1-\gamma}, \quad C_N = \left(\frac{\gamma}{1-\gamma} \cdot \frac{P_N}{P_T}\right)^{-\gamma}$$

$$P = P_T^{\gamma} P_N^{1-\gamma} A, \qquad A \equiv \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}$$

Real income Y/P drives welfare; mis-measuring γ skews welfare comparisons.

8.10.1 Estimation of γ

We divide C_T by C_N and solve for γ:

$$\gamma = \frac{C_T P_T}{C_T P_T + C_N P_N}$$