Consumption, Saving & the Fisher Model

ECON0004: Introductory Econometrics & Microeconomics

UCL

University College London

March 2026

Based on: Lecture notes (Lectures 7.1–7.2); Fisher (1930) *The Theory of Interest*; Deaton (1992) *Understanding Consumption*.

Outline

1. The Keynesian Consumption Function
2. Fisher Two-Period Model
3. Optimal Consumption Choice
4. Discounted Utility and the Euler Equation

Why Does Consumption Matter?

Consumption accounts for roughly 60–70% of GDP in most economies. Understanding what drives it is central to macroeconomic analysis.

Key puzzle: consumption and income diverge over the business cycle. During recessions, income falls sharply, but consumption often falls much less.

Two competing theories:

1. **Keynesian:** consumption depends only on *current* income
2. **Life-cycle / Permanent income:** consumption depends on *lifetime* expected income

The difference has major policy implications.

Socratic question: If consumers base spending on lifetime income rather than current income, what happens to the effectiveness of a one-time tax rebate?

The Keynesian Consumption Function

Keynesian Consumption Function

$$C = c_0 + c_1 Y$$

- $c_0 > 0$ — autonomous consumption (spending when $Y = 0$)
- $0 < c_1 < 1$ — marginal propensity to consume (MPC)

Average propensity to consume (APC):

$$APC = \frac{C}{Y} = \frac{c_0}{Y} + c_1$$

APC **falls** as income increases (since c_0/Y shrinks).

Empirical evidence:

- MPC significantly positive (c_1 statistically significant)
- Cannot reject $c_0 = 0$ (autonomous consumption insignificant)

Problem 1: Consumption smoothing.
The Keynesian model implies consumption fluctuates one-for-one with income. But in data:

- Consumption fluctuates **less** than disposable income in the short run
- Households maintain spending despite temporary income falls (e.g. illness, recession)

Problem 2: Business-cycle anomalies.
During recovery, consumers behave cautiously — spending rises more slowly than income. The Keynesian model cannot explain this asymmetry.

> **Root cause:** The Keynesian model treats consumption as a *static* decision. It ignores that households can **borrow and save** to transfer resources across time.

Post-Keynesian Theories

Consumers plan their consumption based on *long-term income expectations*, not just current income. The theory is rooted in the microeconomics of consumer behaviour.

Key contrast with Keynes:

- Keynes: $C = f(Y_{\text{current}})$
- Life-cycle/Permanent income: $C = f(Y_{\text{lifetime}})$

Credit-constrained households cannot borrow against future income, so they behave like Keynesians — spending tracks current income.

The **Fisher Two-Period Model** is the canonical microeconomic framework for intertemporal consumption choice. It yields testable predictions about how consumption responds to income changes and interest rate changes.

Fisher Two-Period Model

A consumer lives for two periods ($t = 1, 2$) and chooses (C_1, C_2) to maximise lifetime utility subject to a lifetime budget constraint.

Key assumptions:

1. Consumers **maximise lifetime utility** subject to a lifetime budget constraint
2. Non-satiation and diminishing marginal utility: $\partial U / \partial C_1 > 0$, $\partial U / \partial C_2 > 0$, $U_{11}, U_{22} < 0$
3. Borrowing and saving occur at the **same** real interest rate r
4. No uncertainty: all future income is known

Notation:

- Y_1, Y_2 — real disposable income in periods 1 and 2
- W_1 — initial financial wealth (assets carried into period 1)
- S_1 — saving in period 1 ($S_1 < 0$ if borrowing)

Human Wealth and Budget Constraints

Human Wealth

$h_t = \mathbb{P}_t(y)$ — the present value of all future labour income from the perspective of period t.

$$h_1 = Y_1 + \frac{Y_2}{1+r}$$

Period 1 budget constraint:

$$C_1 + S_1 = W_1 + Y_1$$

Resources = wealth plus income; allocation = consumption plus saving.

Period 2 budget constraint:

$$C_2 = Y_2 + (1+r)S_1$$

Resources = period-2 income plus accumulated savings/debt (S_1 carried forward at rate r).

If $S_1 < 0$ (borrowing in period 1), the consumer *repays* $(1+r)|S_1|$ in period 2, so C_2 is reduced

Intertemporal Budget Constraint (IBC)

Eliminate S_1 by substituting Period 1 constraint into Period 2:

Step 1. From Period 1: $S_1 = W_1 + Y_1 - C_1$. Substitute into Period 2: $C_2 = Y_2 + (1+r)(W_1 + Y_1 - C_1)$.

Step 2. Rearrange to the **present-value form**:

$$\boxed{C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} + W_1}$$

Step 3. Or in **future-value form**:

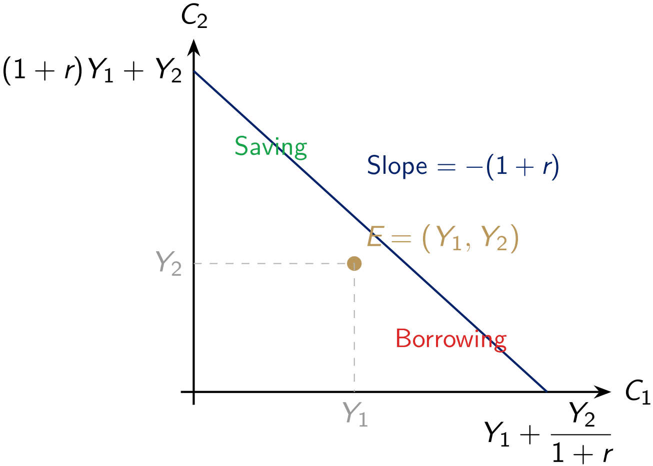
$$(1+r)C_1 + C_2 = (1+r)Y_1 + Y_2 + (1+r)W_1$$

IBC: PV of lifetime consumption = PV of lifetime resources. The consumer cannot consume more than their lifetime wealth in present value.

IBC: Graphical Representation

Set $W_1 = 0$ for simplicity. Rearrange IBC to slope-intercept form:

$$C_2 = (1+r)(Y_1 - C_1) + Y_2 = -(1+r)C_1 + \left[(1+r)Y_1 + Y_2\right]$$



The constrained problem:

$$\max_{C_1, C_2} U(C_1, C_2) \quad \text{s.t.} \quad C_2 = (1+r)(Y_1 - C_1) + Y_2$$

Lagrangian:

$$\mathcal{L}(C_1, C_2, \lambda) = U(C_1, C_2) - \lambda\left[C_2 - (1+r)(Y_1 - C_1) - Y_2\right]$$

Optimality condition (MRS = MRT):

$$1 + r = MRS_{12} = \frac{MU_1}{MU_2} = \frac{\partial U/\partial C_1}{\partial U/\partial C_2}$$

At the optimum, the marginal rate of substitution between today's and tomorrow's consumption equals the gross interest rate $(1+r)$. The consumer equates the marginal cost of saving (foregone current consumption) to the marginal benefit (extra future consumption).

Consumption Smoothing: Income Changes

What happens when income rises (in either period)?

An income increase in Period 1 or Period 2 raises the **present value of lifetime resources**, shifting the IBC outward in parallel.

Increase in Y_1:

- PV of resources rises
- Consumer saves part of the gain to finance higher C_2
- **Both C_1 and C_2 increase**
- This is **consumption smoothing**

Increase in Y_2:

- Consumer borrows against future income
- **Both C_1 and C_2 increase**

> Life-cycle: timing of income is **irrelevant** — only lifetime PV matters. Keynes: only Y_1 matters.

A rise in r **pivots** the IBC inward at the endowment point $E = (Y_1, Y_2)$ (the budget line rotates, Y-intercept rises, X-intercept falls).

Substitution Effect: C_1 becomes more expensive (higher opportunity cost). $\Rightarrow$ Decrease C_1, increase C_2.

Income Effect (depends on position):

- **Saver** $(S_1 > 0)$: real wealth rises $\Rightarrow$ increase both C_1 and C_2
- **Borrower** $(S_1 < 0)$: real wealth falls $\Rightarrow$ decrease both C_1 and C_2

	C_1	C_2
Saver	SE $\downarrow$, IE $\uparrow$ $\Rightarrow$ **ambiguous**	$\uparrow$
Borrower	SE $\downarrow$, IE $\downarrow$ $\Rightarrow$ **falls**	ambiguous

With T periods, the consumer maximises the present value of lifetime utility:

$$V = \sum_{t=0}^{T} U_t(c_t) = U_0(c_0) + U_1(c_1) + \cdots + U_T(c_T)$$

subject to the sequence of intertemporal budget constraints.

Optimal condition between adjacent periods:

$$MRS_{t,t+1} = \frac{MU_t}{MU_{t+1}} = \frac{\mathrm{d}V/\mathrm{d}c_t}{\mathrm{d}V/\mathrm{d}c_{t+1}} = 1 + r$$

> The multi-period Fisher condition says: at the optimum, the consumer is indifferent between consuming one extra unit today and saving it for tomorrow. If $MU_t/(MU_{t+1}) > 1 + r$, the consumer should consume more today.

Discounted Utility Model ($\beta < 1$)

$$U_t = \frac{1}{(1+\delta)^t} \cdot \frac{1}{\beta} c_t^{\beta}$$

- $\delta > 0$ — subjective discount rate; higher $\delta \Rightarrow$ more impatient
- $\beta < 1$ — curvature parameter; IES $= 1/(1-\beta) > 0$

Notation note: Here β is the IES curvature parameter, *not* a regression coefficient. Similarly, δ here is the discount rate, not the demographic coefficient of Lectures 6/9.

Intuition:

- Higher $\delta \Rightarrow$ lower IES $\Rightarrow$ consumption growth falls (impatient consumers spend more today)
- E.g., if $\delta = 0.05$, $\beta = 0.5$ (IES $= 2$): a 1pp rise in r raises consumption growth by 2pp

Proof that IES $= 1/(1-\beta)$:

The curvature of $u(c) \propto c^\beta/\beta$ governs intertemporal substitution:

$$\varepsilon = -\frac{u'(c)}{c \cdot u''(c)} = -\frac{\frac{1}{(1+\delta)^t} c_t^{\beta-1}}{\frac{1}{(1+\delta)^t}(\beta-1)c_t^{\beta-2} \cdot c} = \frac{1}{1-\beta}$$

IES $= 1/(1-\beta)$:

- $\beta \to 0$: log utility ($U \approx \ln c$), IES $= 1$
- $\beta < 0$: concave utility, IES < 1 (strong preference for smooth consumption)
- $\beta \to 1$: linear utility, IES $\to \infty$ (indifferent between timing)

Euler Equation Derivation

Optimality: $MU_t = (1+r)\, MU_{t+1}$, where $MU_t = \frac{1}{(1+\delta)^t} c_t^{\beta-1}$.

Step 1. Substitute: $\frac{1}{(1+\delta)^t} c_t^{\beta-1} = (1+r) \frac{1}{(1+\delta)^{t+1}} c_{t+1}^{\beta-1}$

Step 2. Rearrange: $\dfrac{1+r}{1+\delta} = \left(\dfrac{c_{t+1}}{c_t}\right)^{1-\beta}$

Step 3. Let $g = c_{t+1}/c_t - 1$. Take logs; use $\ln(1+x) \approx x$:

$$\boxed{g \approx \frac{1}{1-\beta}(r-\delta)}$$

Euler equation: consumption growth rises with r and falls with δ. IES $= 1/(1-\beta)$ is the gradient in the regression $\Delta \ln c_t = \alpha + \frac{1}{1-\beta} r_t + \varepsilon_t$.

Regression form:

$$\Delta \ln c_t = \underbrace{-\frac{\delta}{1-\beta}}_{\text{intercept}} + \underbrace{\frac{1}{1-\beta}}_{\text{IES}} r_t + \varepsilon_t$$

Empirical implications:

- **Higher r:** consumption growth increases
- **Higher δ:** consumption growth decreases

Hypothesis tests:

Null	**Equivalent test on regression**
$H_0 : \beta = 0$ (log utility)	Gradient $= 1/(1-\beta) = 1$
$H_1 : \beta \neq 0$	Gradient $\neq 1$

Life-cycle assumptions: No unforeseen income shocks; access to credit markets and saving opportunities; agents maximise PV of lifetime utility.

Lecture 7 Summary

1. **Keynesian model:** $C = c_0 + c_1 Y$. Simple but ignores intertemporal substitution and consumption smoothing.
2. **Fisher IBC:** $C_1 + C_2/(1+r) = Y_1 + Y_2/(1+r) + W_1$. PV of lifetime consumption $=$ PV of lifetime resources.
3. **Optimality:** MRS $= 1 + r$. Consumer equates marginal utilities across periods weighted by the interest factor.
4. **Consumption smoothing:** Income increases in *either* period raise both C_1 and C_2. Timing of income is irrelevant (absent credit constraints).
5. **Euler equation:** $g \approx \frac{1}{1-\beta}(r - \delta)$. IES $= 1/(1-\beta)$; impatience δ lowers consumption growth. Testable via OLS regression of $\Delta \ln c_t$ on r_t.
6. **Life-cycle assumptions:** No income shocks, access to credit markets, agents maximise PV of lifetime utility.