Consumer Demand

ECON0004: Introductory Econometrics & Microeconomics

UCL

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Based on: Lecture notes (Lecture 6); Deaton & Muellbauer (1980) *Economics and Consumer Behaviour*.

Outline

What is Consumer Demand Theory?

Central question: How do households allocate scarce income across goods?

Consumer Demand Theory

The study of how households allocate their income across goods in a given period, given prices and preferences.

Two fundamental questions:

- What determines **which** goods consumers buy?
- How does spending on each good respond to changes in income or prices?

Suppose there are J goods. Household i's demand vector $(q_{i1}, q_{i2}, \ldots, q_{iJ})$ depends on its income y_i, prices $(p_{i1}, \ldots, p_{iJ})$, and preferences.

Socratic question: Why might knowing how demand responds to prices matter for a government setting an excise tax on alcohol?

Classifying Goods by Income Response

When income y_i rises, how does demand for good j change?

Good type	Demand response	Budget share
Normal good	q_{ij} rises with y_i	—
Necessity	q_{ij} rises, but slowly	Falls as y_i rises
Luxury	q_{ij} rises, proportionally more	Rises as y_i rises
Inferior good	q_{ij} falls with y_i	Falls as y_i rises

Example

Examples: Foreign holidays (luxury); bread (necessity); instant noodles (inferior good for higher-income households).

Why Measure Proportional Responses?

Raw derivatives $\partial q_{ij}/\partial y_i$ are hard to interpret across goods with different units (kilograms, litres, trips).

Elasticity

An elasticity measures the *percentage change* in one variable in response to a *one-percent change* in another — a unit-free measure of sensitivity.

General formula:

$$\xi_{x,z} = \frac{\partial x}{\partial z} \cdot \frac{z}{x} = \frac{\partial \ln x}{\partial \ln z}$$

Economists prefer elasticities because they are **dimensionless** and directly comparable across goods, countries, and time periods.

Income Elasticity of Demand (YED)

Income Elasticity of Demand

$$\xi_{q_j,y} = \frac{\partial q_{ij}}{\partial y_i} \cdot \frac{y_i}{q_{ij}} = \frac{\partial \ln q_{ij}}{\partial \ln y_i}$$

Sign / Range	Good type	Interpretation
$\xi < 0$	Inferior good	Demand falls with income
$0 < \xi < 1$	Necessity	Demand rises, less than proportionally
$\xi > 1$	Luxury	Demand rises, more than proportionally

Example

Example: Food (necessity, $\xi \approx 0.3$–0.5) vs. foreign holidays (luxury, $\xi > 1$). A 10% income rise $\Rightarrow$ 3–5% more food spending but 15–20% more holiday spending.

Price Elasticity of Demand (PED)

Price Elasticity of Demand

$$\xi_{q_j,p_j} = \frac{\partial q_{ij}}{\partial p_{ij}} \cdot \frac{p_{ij}}{q_{ij}} = \frac{\partial \ln q_{ij}}{\partial \ln p_{ij}}$$

Percentage change in quantity demanded per one-percent rise in the *own* price.

Value	Label	Total expenditure effect
$\xi > 0$	Giffen good (rare)	Violates Law of Demand
$\xi = -1$	Unitary elastic	Expenditure unchanged
$\xi < -1$	Price elastic	Expenditure falls as price rises
$-1 < \xi < 0$	Price inelastic	Expenditure rises as price rises

Socratic question: A government wants to reduce tobacco consumption. Will an excise tax be more effective if demand is elastic or inelastic? What does the answer imply for tax revenue?

Cross-Price Elasticity of Demand (XED)

Cross-Price Elasticity of Demand ($k \neq j$)

$$\xi_{q_j, p_k} = \frac{\partial q_{ij}}{\partial p_{ik}} \cdot \frac{p_{ik}}{q_{ij}} = \frac{\partial \ln q_{ij}}{\partial \ln p_{ik}}$$

Sign	Magnitude	Goods are ...
$-$	$\lvert\xi\rvert \gtrless 1$	Complements (weak or strong)
0	—	Unrelated goods
$+$	$\lvert\xi\rvert \gtrless 1$	Substitutes (weak or strong)

Sign symmetry: If two goods are complements, both XEDs carry the same sign. (Magnitudes need not be equal.)
Example: Coffee price $+10\%$ $\Rightarrow$ tea demand $+5\%$; XED $= +0.5$ (substitutes). The XED of coffee w.r.t. tea prices is also positive.

Decomposing a Price Change

When the price of good j rises, two effects operate simultaneously:

Substitution Effect (SE)

Good j becomes relatively more expensive compared to other goods. $\Rightarrow$ Consumers substitute away from j *regardless of good type*.

Income Effect (IE)

The price rise reduces real purchasing power. Direction depends on good type:
Normal: q_j falls;
Inferior: q_j rises.

Overall effect:

- **Normal good:** SE and IE reinforce $\Rightarrow$ q_j falls *(Law of Demand)*
- **Inferior good:** IE partly offsets SE; q_j usually still falls
- **Giffen good:** IE $>$ SE $\Rightarrow$ q_j *rises* — violates Law of Demand

Giffen Goods

Giffen Good

An inferior good for which the income effect dominates the substitution effect, so quantity demanded *increases* when its price rises.

Conditions required:

1. Good must be **inferior** (IE works against SE)
2. Good must take a **large share of the budget** (so IE is large)
3. The income effect must **outweigh** the substitution effect

Example

Classic example: staple foods (bread, potatoes) for very poor households. If bread becomes more expensive, the household cannot afford other foods, so it actually buys *more* bread as a cheap calorie source. Giffen goods are rare in practice — Jensen & Miller (2008) document evidence from rural China.

Demographic Demand Shifters

Beyond price and income, consumer demand is also shaped by **preferences**. Preferences vary systematically with household characteristics.

Demographic characteristics

- Age, gender, ethnicity, residency
- Affect inherent tastes for goods

Household size

- Larger households have lower per-person living standard for a given budget
- Economies of scale in some goods (heating, housing)

Family composition

- Adults vs. children have different consumption needs
- Male-headed vs. female-headed households differ in spending patterns

> In empirical demand systems, Z_i is a vector of demographic characteristics included to control for preference heterogeneity.

From Theory to Empirics

We have defined elasticities conceptually and decomposed price effects into substitution and income components.

The remaining question: How do we **estimate** these elasticities from household data?

We need to choose a **functional form** for the demand equation that:

1. Is estimable by OLS
2. Makes elasticity interpretation easy
3. Is flexible enough to fit data well
4. Can be tested against consumer theory

Socratic question: If a household spends nothing on a good (zero expenditure), what happens if we try to take $\ln q_{ij}$ as the dependent variable? How might we work around this?

Measuring Elasticities: Why and How?

Practical reasons to estimate elasticities empirically:

- Firms set prices based on PED to maximise revenue
- Governments need PED and XED to design effective excise duties
- YED helps governments forecast revenue from income taxes

Choosing a functional form. We aim for:

1. Ease of interpretation
2. Flexibility in describing data
3. Ability to test restrictions suggested by consumer theory
4. Accommodation of zero expenditure (households that do not purchase a good)

> Two main specifications: the **double-log** (constant-elasticity) model and the **budget-share** (linear) model.

Double-Log Demand Function

Also called the **log-linear** or **constant-elasticity** specification:

$$\ln q_{ij} = \alpha + \beta \ln y_i + \sum_{k=1}^{J} \gamma_k \ln p_{ik} + \delta Z_i + \varepsilon_i$$

- q_{ij} — quantity of good j consumed by household i
- y_i — total income/expenditure (assuming MPC $= 1$, no saving)
- p_{ik} — price of good k for household i
- Z_i — demographic characteristics vector

Constant elasticity property:

$$\xi_{q_j,y} = \frac{\partial \ln q_{ij}}{\partial \ln y_i} = \beta, \qquad \xi_{q_j,p_j} = \gamma_j, \qquad \xi_{q_j,p_k} = \gamma_k \ (k \neq j)$$

The coefficient γ_j on $\ln p_{ij}$ measures **own-price elasticity**; all other γ_k measure **cross-price elasticities**. Parameters are elasticities directly — easy to interpret.

Double-Log: Advantages and Disadvantages

Advantages

- Coefficients are directly interpretable as elasticities
- Simple OLS estimation after taking logs
- Log transformation reduces heteroscedasticity

Disadvantages

- Cannot include households with **zero expenditure** on good j ($\ln 0$ is undefined)
- Elasticities are **constant** — the same at every income and price level
- This constant-elasticity assumption may be unrealistic

Socratic question: A household dataset includes many households that spend nothing on alcohol. Why does this create a problem for the double-log model, and how might the budget-share model help?

Budget Share Specification

Define the **budget share** of good j for household i:

$$w_{ij} = \frac{p_{ij} q_{ij}}{y_i} \quad \text{(fraction of total spending on good } j\text{)}$$

The budget-share model replaces $\ln q_{ij}$ with w_{ij} on the LHS:

$$w_{ij} = \alpha + \beta \ln y_i + \sum_{k=1}^{J} \gamma_k \ln p_{ik} + \delta Z_i + \varepsilon_i$$

> The RHS is **identical** to the double-log model — only the dependent variable changes. This small change has large implications for what the coefficients mean.

Budget Share: Elasticity Derivations

Income Elasticity (YED):

Step 1. Rearrange $w_{ij} = p_{ij}q_{ij}/y_i$ to get $p_{ij}q_{ij} = y_i w_{ij}$. Differentiate w.r.t. y_i: $p_{ij}\,\partial q_{ij}/\partial y_i = w_{ij} + (\partial w_{ij}/\partial y_i)\, y_i$.

Step 2. From the model, $\beta = \partial w_{ij}/\partial \ln y_i = y_i\,\partial w_{ij}/\partial y_i$. Substitute: $p_{ij}\,\partial q_{ij}/\partial y_i = w_{ij} + \beta$.

Step 3. Multiply both sides by $y_i/(p_{ij}q_{ij}) = 1/w_{ij}$:

$$\xi_{q_j,y} = \frac{y_i}{q_{ij}} \cdot \frac{\partial q_{ij}}{\partial y_i} = 1 + \frac{\beta}{w_{ij}}$$

YED $= 1 + \beta/w_{ij}$. Luxury good if $\beta > 0$; necessity if $\beta < 0$. Unlike the double-log, YED **varies with the budget share** w_{ij}.

Budget Share: PED and XED

Own-price elasticity (PED):

$$\xi_{q_j,p_j} = \gamma_j / w_{ij} - 1$$

- Price inelastic when $\gamma_j > 0$ (budget share rises as price rises)
- Price elastic when $\gamma_j < 0$ (budget share falls as price rises)

Cross-price elasticity (XED, $k \neq j$):

$$\xi_{q_j,p_k} = \gamma_k / w_{ij}$$

- Substitute for good k if $\gamma_k > 0$
- Complement to good k if $\gamma_k < 0$

Interpretation: "A 1% change in the price of good k changes the budget share of good j by γ_k percentage points."

Comparison: Double-Log vs. Budget Share

	Double-log	Budget share
Dependent variable	$\ln q_{ij}$	w_{ij}
YED	β (constant)	$1 + \beta / w_{ij}$ (varies)
PED	γ_j (constant)	$\gamma_j / w_{ij} - 1$ (varies)
Zero expenditure	Not allowed	Allowed ($w_{ij} = 0$ is fine)
Elasticity constancy	Yes — unrealistic	No — more realistic
Testing theory	Harder	Easier

Non-constant elasticities in the budget-share model: A coefficient of 0.01 on $\ln p_k$ represents a larger proportional change for a household with $w_{ij} = 0.01$ (1% share) than one with $w_{ij} = 0.99$. This is a natural feature of spending behaviour.

Lecture 6 Summary

1. **Goods classification:** Normal (necessity/luxury) vs. inferior. Defined by the sign and magnitude of the income elasticity (YED).
2. **Elasticities** measure proportional responses: YED, PED, XED. All can be read as $\partial \ln q / \partial \ln(\text{variable})$.
3. **Price decomposition:** SE always reduces q_j when price rises; IE direction depends on good type. Giffen goods arise when IE $>$ SE for an inferior good.
4. **Double-log model:** $\ln q_{ij} = \alpha + \beta \ln y_i + \sum_k \gamma_k \ln p_{ik} + \delta Z_i + \varepsilon_i$. Coefficients are constant elasticities. Cannot handle zero expenditure.
5. **Budget-share model:** $w_{ij} = \alpha + \beta \ln y_i + \sum_k \gamma_k \ln p_{ik} + \delta Z_i + \varepsilon_i$. YED $= 1 + \beta / w_{ij}$. Allows zero expenditure; elasticities vary with budget share.